# Luminosity distance dispersion in Swiss-cheese cosmology as a function of the hole size distribution

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# A Bird's Eye View of Cosmology

- Cosmology is an observational science!
- Goal:
  - (i) Infer the components of the universe
  - (ii) Study how those components evolve
- Usually, one does this by comparing observations to a *model*, constructed from theory.
  - A canonical example: luminosity distance-redshift relation

Credit: Huntington Library

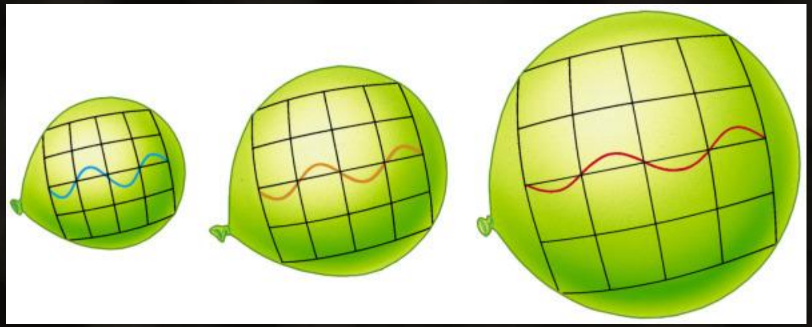
# Luminosity Distance and Redshift

## Luminosity distance ($D_L$)

- Distances inferred from apparent brightness of sources with known luminosity (standard candles)

## Redshift ($z$)

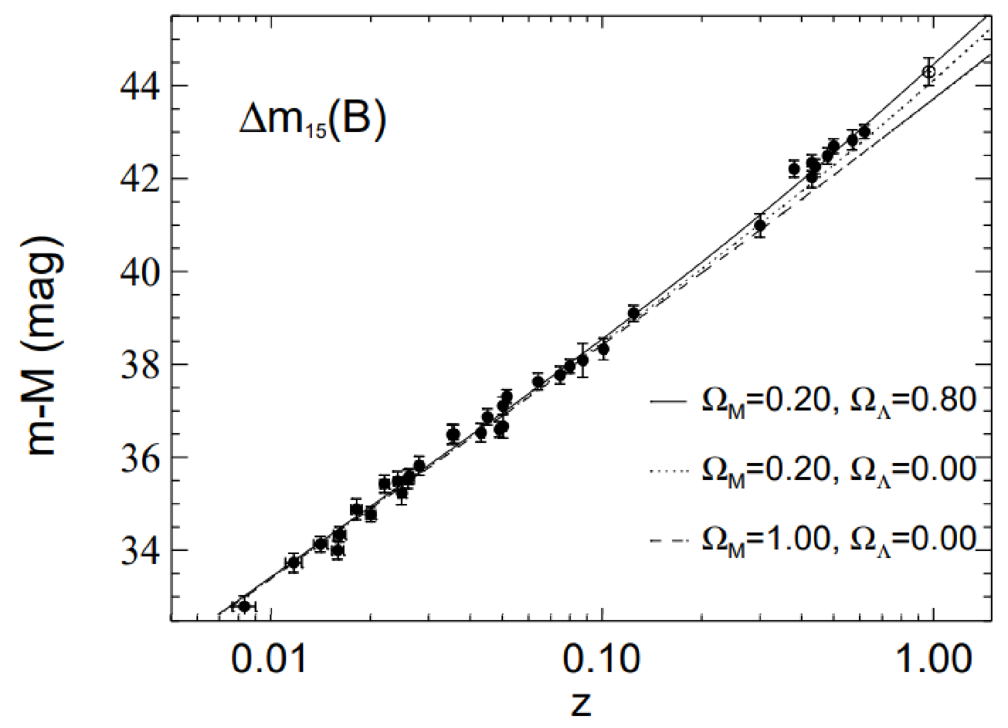
- The fractional change of the photon's wavelength
- *Only talking about cosmological redshift*



# $D_L - z$ Relation

- We observe these $(D_L, z)$ →
- Then we fit the theory $D_L(z|\theta)$
  - Infer $\theta = \{\Omega_\mathrm{M}, H_0\}$

- Observation: Supernova Type Ia
- Theory: Friedmann-Robertson-Walker Metric




Credit: Riess et al. (1998)

# Friedmann-Robertson-Walker (FRW) Metric

**Isotropy**: the universe looks the same *in every direction*

**Homogeneity**: the universe looks the same *at every location*

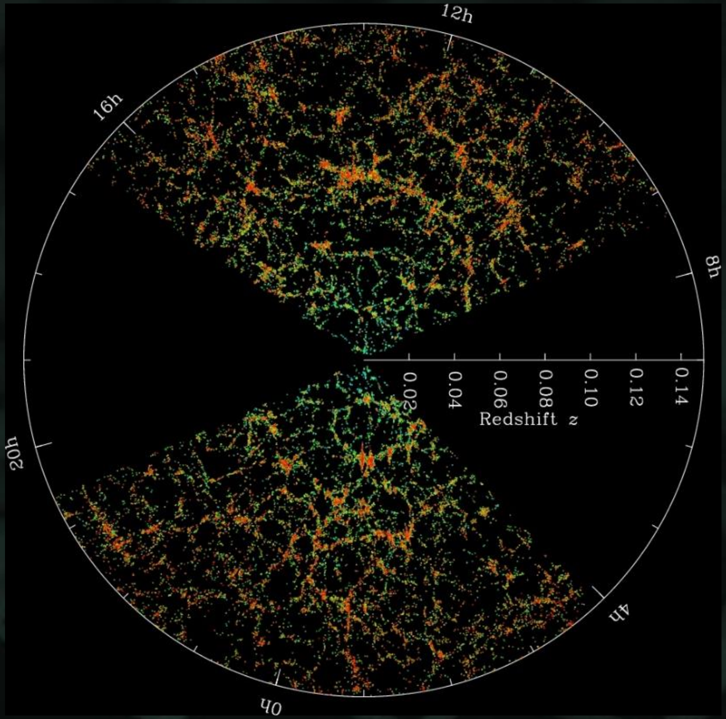
## FRW Metric

$$ds^2 = -dt^2 + a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega \right]$$

- From this, we can derive $D_L - z$ relation

# Inhomogeneity

- Galaxy surveys & simulations observe *cosmic voids*

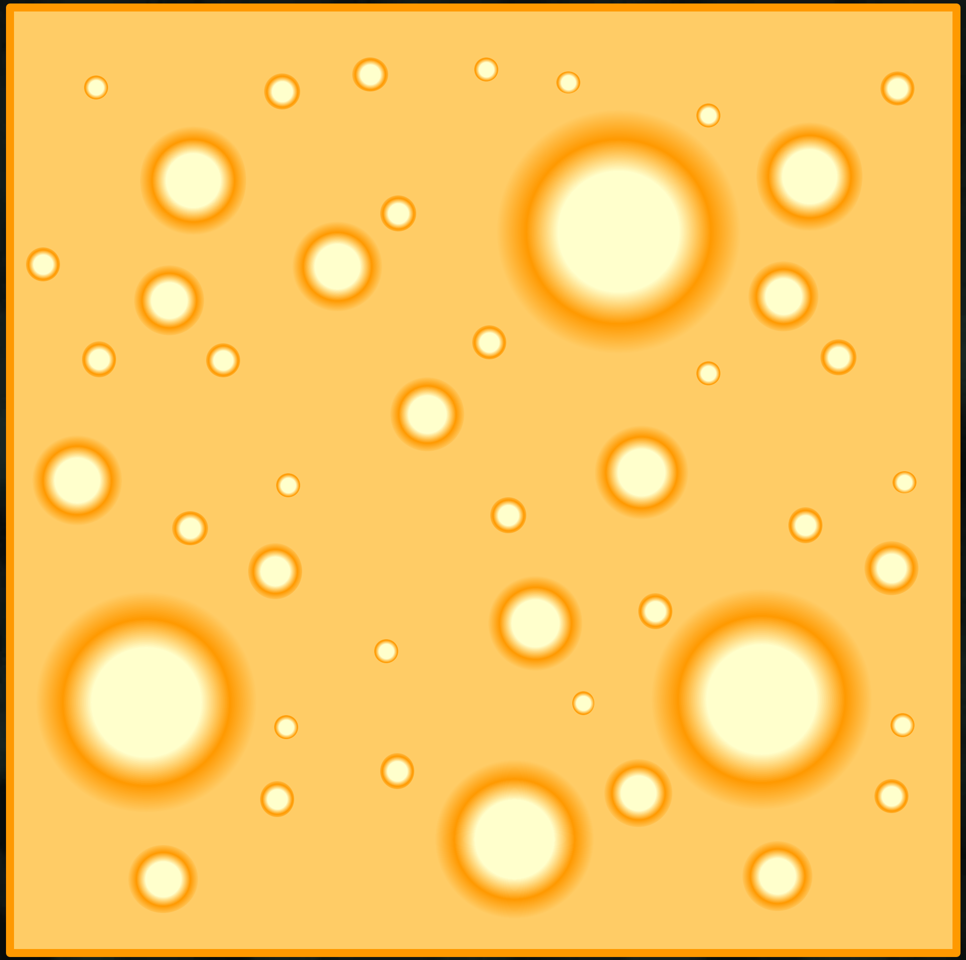



Credit: SDSS

- Metric deviates from FRW at $\leq$ homogeneity scale.
- $(D_L, z)$ varies from one line of sight to the next, depending on inhomogeneities it passes through.
- $D_L$ and $z$ are “fuzzy”.
  - Is this enough to solve various problems in cosmology? Dark energy? $H_0$ tension?
- How do we investigate this?
  - Surveys and simulations are limited by resolution

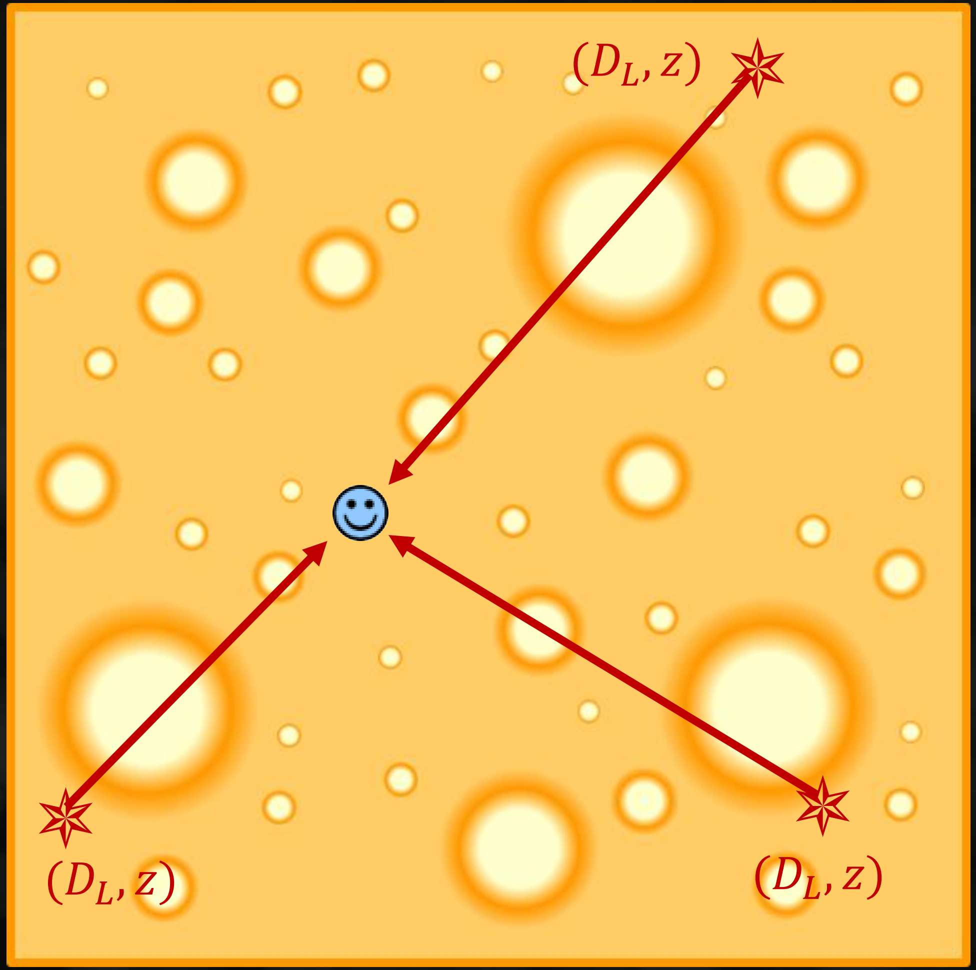
# Swiss-cheese Models

- Homogeneous background w/ spherical voids
- Spherical symmetry → exact solution of GR
- Past works: $\geq$ 10 Mpc holes
- This work:
  - (Hypothetical) small holes
  - power law hole size distribution



# Method

1. Construct a Swiss-cheese universe.
2. Propagate light rays through the holes.
3. Calculate $D_L$ and $z$ from properties of light.



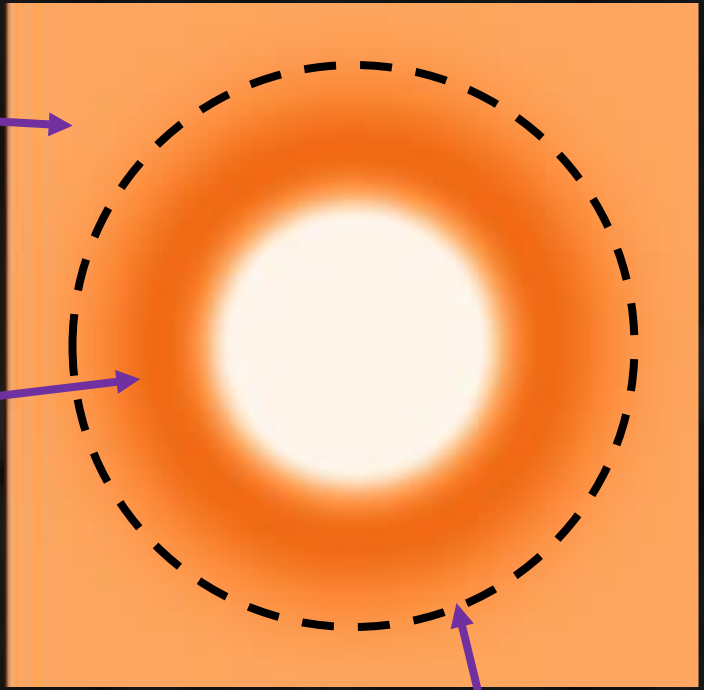
# How to make Swiss-cheese

"Cheese" : FRW metric

$$ds^2 = -dt^2 + a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega \right]$$

"Hole" : Lemaitre-Tolman-Bondi (LTB) metric

$$ds^2 = -dt^2 + a(t,r)^2 \left\{ \left[ 1 + \frac{ra'(t,r)}{a(t,r)} \right]^2 \frac{dr^2}{1 - k(r)r^2} + r^2 d\Omega \right\}$$



Junction Conditions

Constraints: avoid…
- Density divergence/shell crossings
- Blueshift divergence

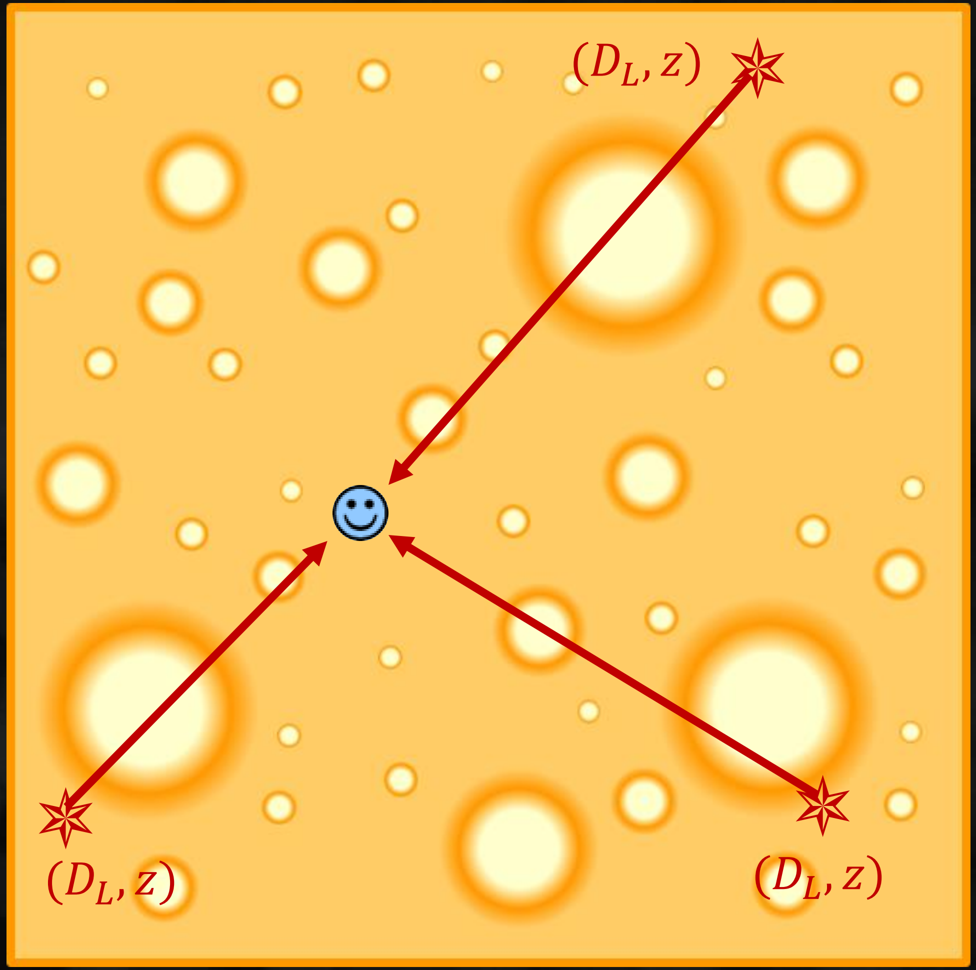
# Light Propagation

- Geodesic Equations

  e.g. $\frac{dp^t(s)}{ds} = -\frac{(\dot{a}+r\dot{a}')(a+ra')}{1-kr^2}(p^r)^2 - \frac{\dot{a}L^2}{r^2a^3}$
- Optical Equations

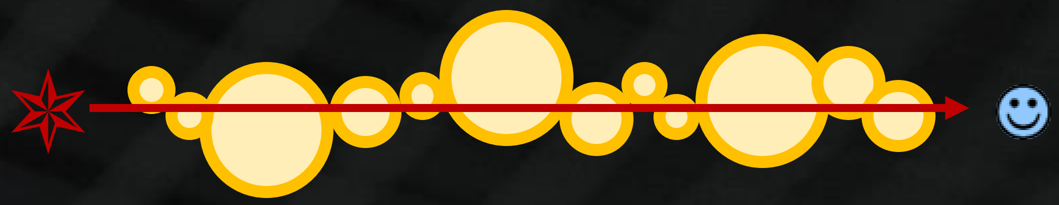
  e.g. $\frac{d\theta}{ds} = -\frac{1}{2}\theta^2 - 2(\sigma_+^2 + \sigma_-^2) + 2\omega^2 + R_{\alpha\beta}p^\alpha p^\beta$

- Relates light properties at source & obs.
- Integrate these ODEs numerically
- Calculate observables $(D_L, z)$

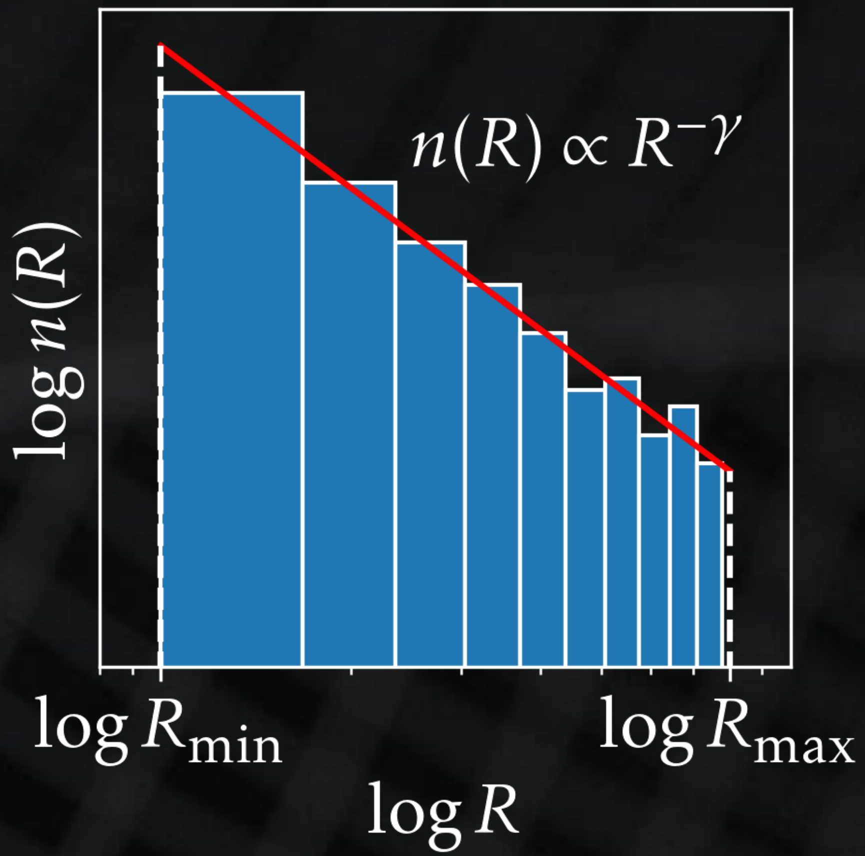


# Hole size distributions

- Light beam encounters holes successively

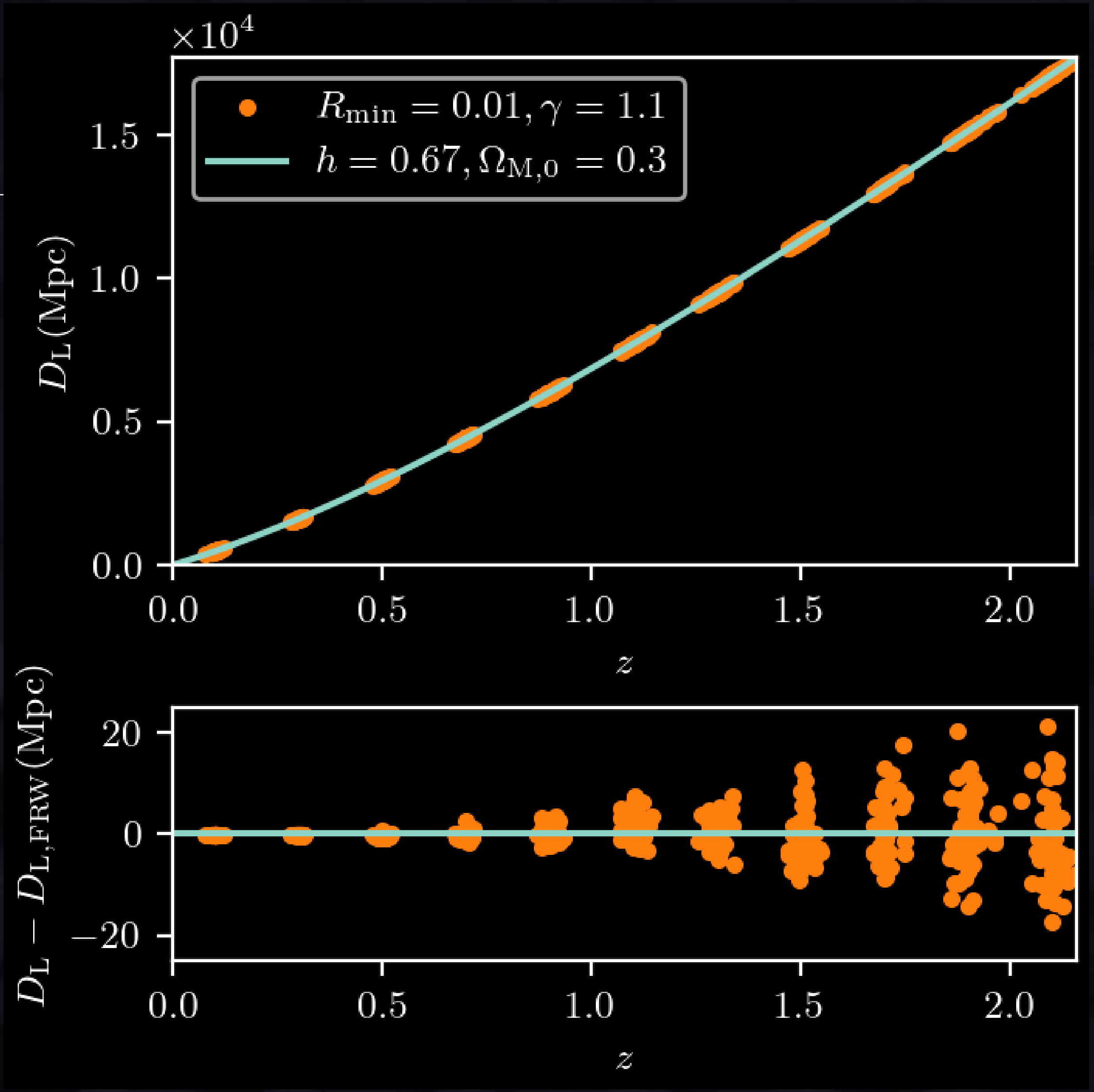


- Sample hole size from power law PDF
  - Minimum hole size $R_{min}$
  - Logarithmic slope $\gamma$



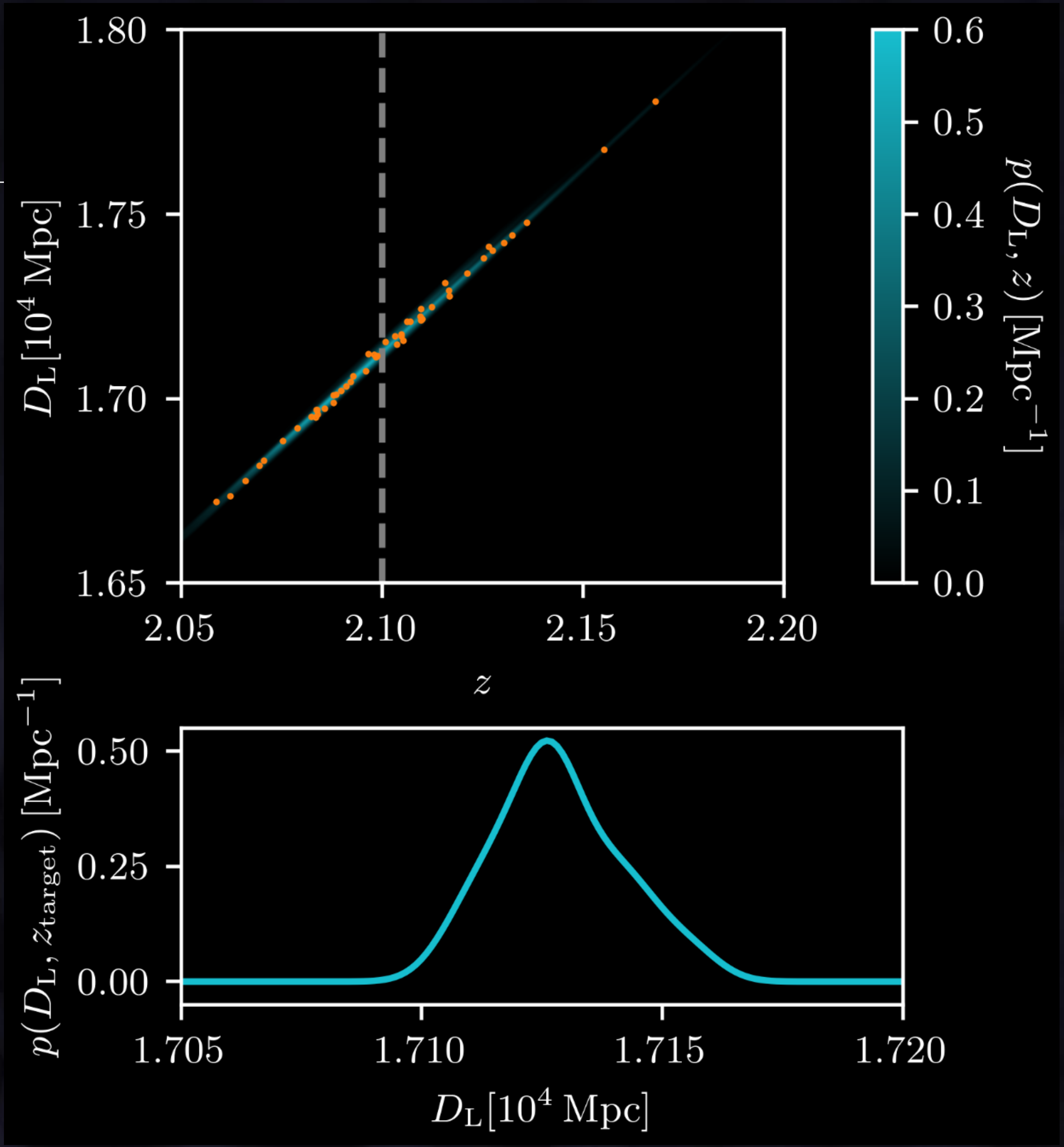
# $D_L - z$ dispersion

- Qualitatively, $(D_L, z)$ lie around the "cheese FRW" $D_L(z)$.
- Dispersion growing with $z$
  - How do we quantify this?
  - Subtracting FRW $D_L(z)$ might contaminate the dispersion



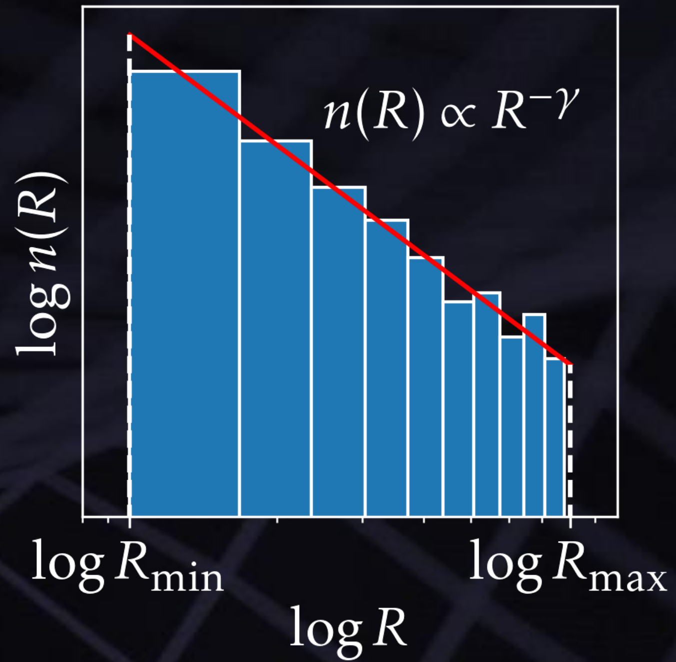
# Measuring $\sigma_{D_L}$

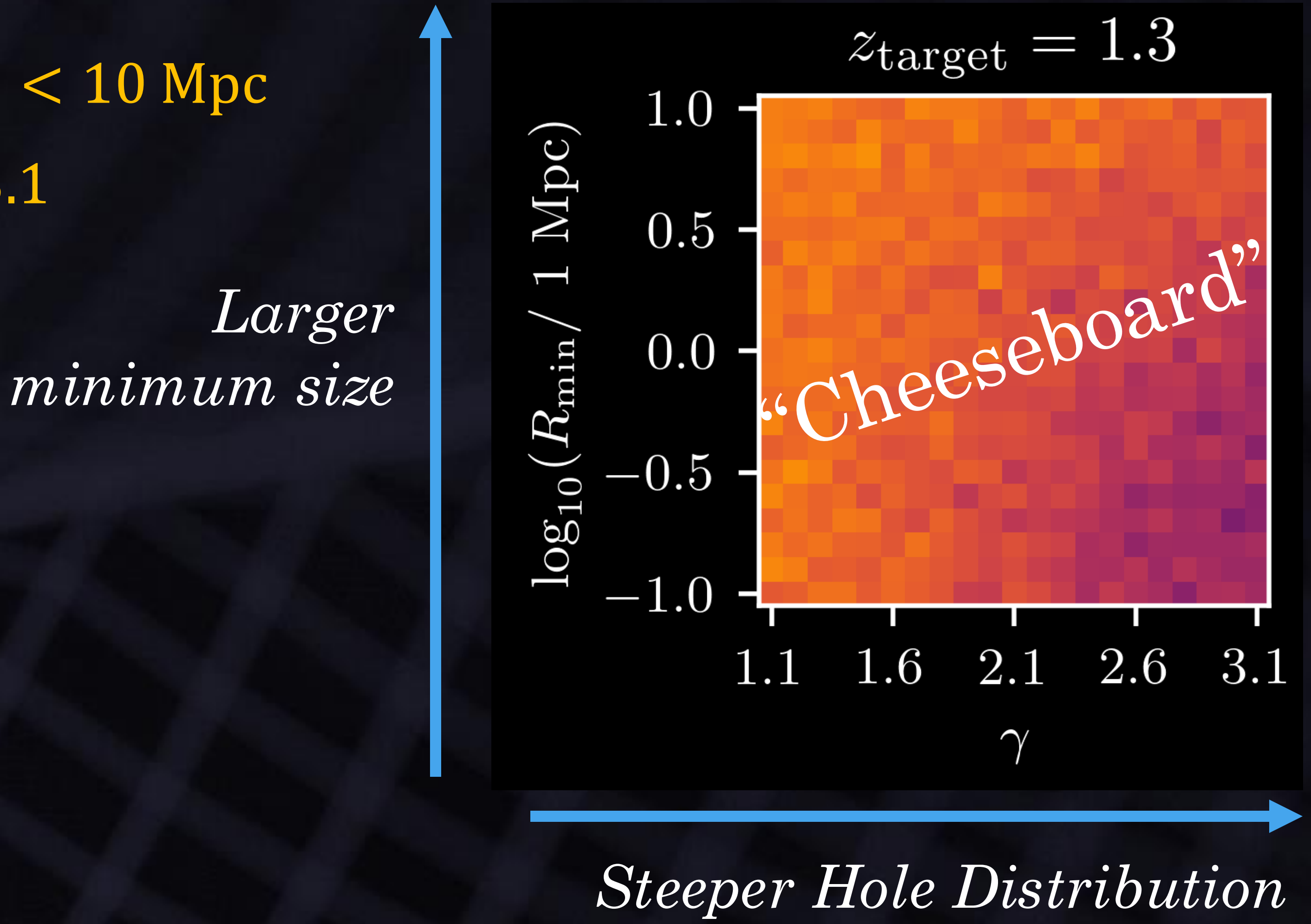
- Generate a sample of 50 $(D_L, z)$ around $z_{\text{target}}$
- Kernel density estimate $p(D_L, z)$
- Calculate width of $p(D_L, z_{\text{target}})$
- Do this for many Swiss-cheese parameters



# Parameter space

- Minimum size: $0.1\ \text{Mpc} < R_{\text{min}} < 10\ \text{Mpc}$
- Logarithmic slope: $1.1 < \gamma < 3.1$

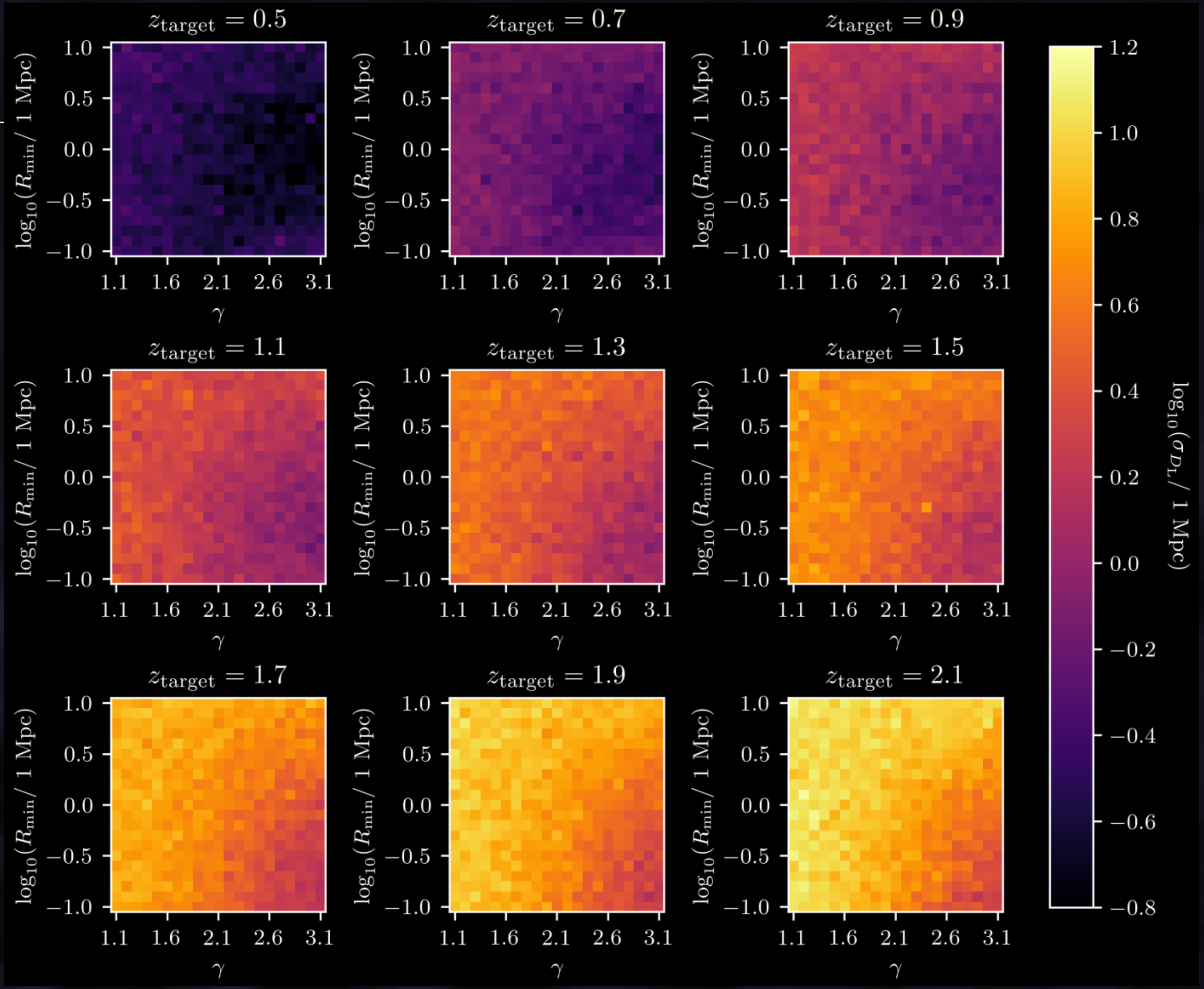


# $D_L$ dispersion

- $\sigma_{D_L}$ decrease towards lower right
- Large $\gamma$, small $R_{\min}$ → Largest fraction of small holes
- Nonlinear least-squares gives

$$\sigma_{D_L} = (2.19\ \mathrm{Mpc})\ z^{2.25}\left(\frac{R_{\min}}{24\ \mathrm{Mpc}}\right)^{0.157(\gamma-1.16)}$$



# Conclusion & Outlook

- $(D_L, z)$ varies from one line of sight to another because it depends on the inhomogeneous metric in between source and observer
- Using a Swiss-cheese model of cosmology, we estimate $D_L$ dispersion

$$\sigma_{D_L} \sim (2.19\ \text{Mpc})\ z^{2.25} \left( \frac{R_{\text{min}}}{24\ \text{Mpc}} \right)^{0.157(\gamma - 1.16)}$$

- Outlook
  - There are other sources of dispersion: void shape/orientation, profile, etc.
  - Understanding this is important for the era of precision cosmology!
- Paper on the verge of submission – stay tuned!