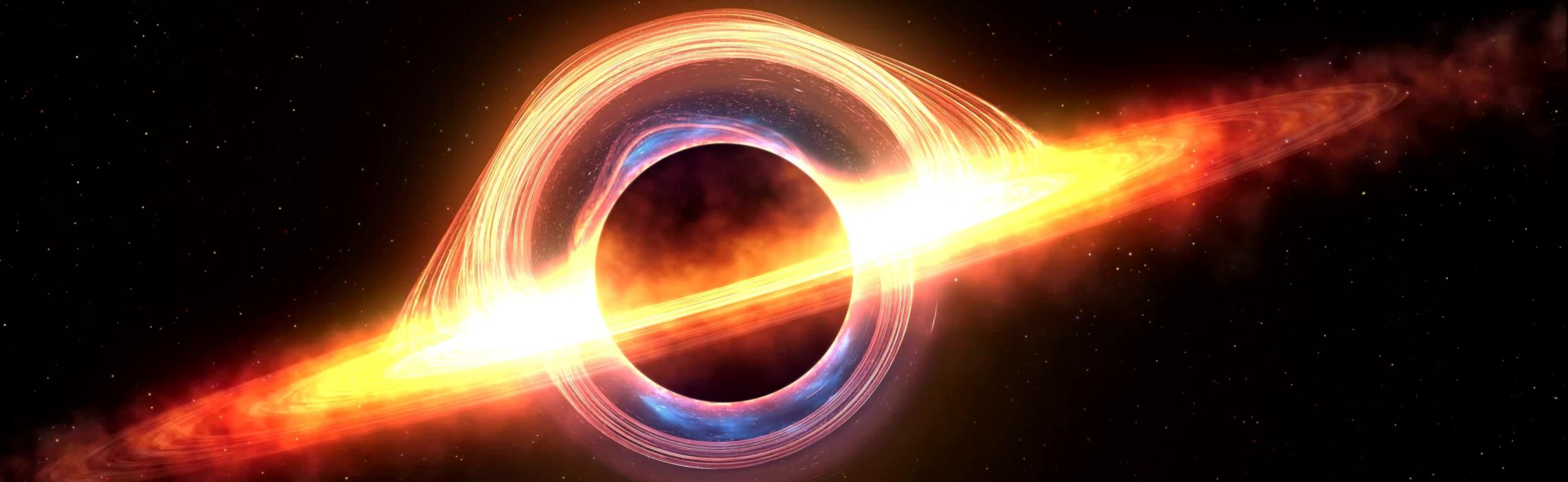
Light rings of nonsingular ultracompact objects sourced by nonlinear electrodynamics



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30 YEARS OF GRAVITY RESEARCH IN AUSTRALASIA
PAST REFLECTIONS AND FUTURE AMBITIONS
CANBERRA, 2-3 SEPTEMBER 2024

Ultracompact Objects

Compact enough to have a light ring!

Cardoso, Pani: 1904.05363

Murk: 2210.03750

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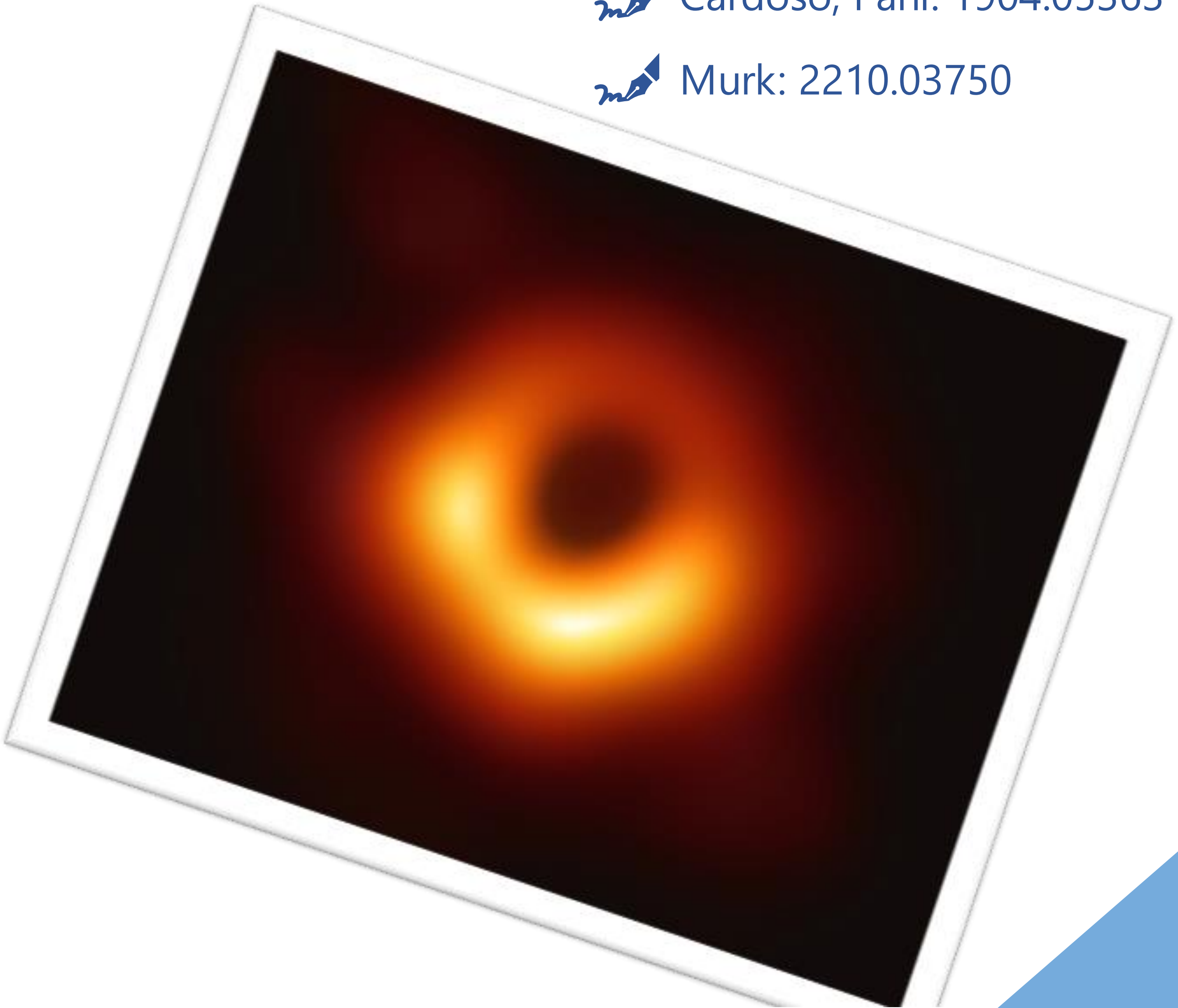

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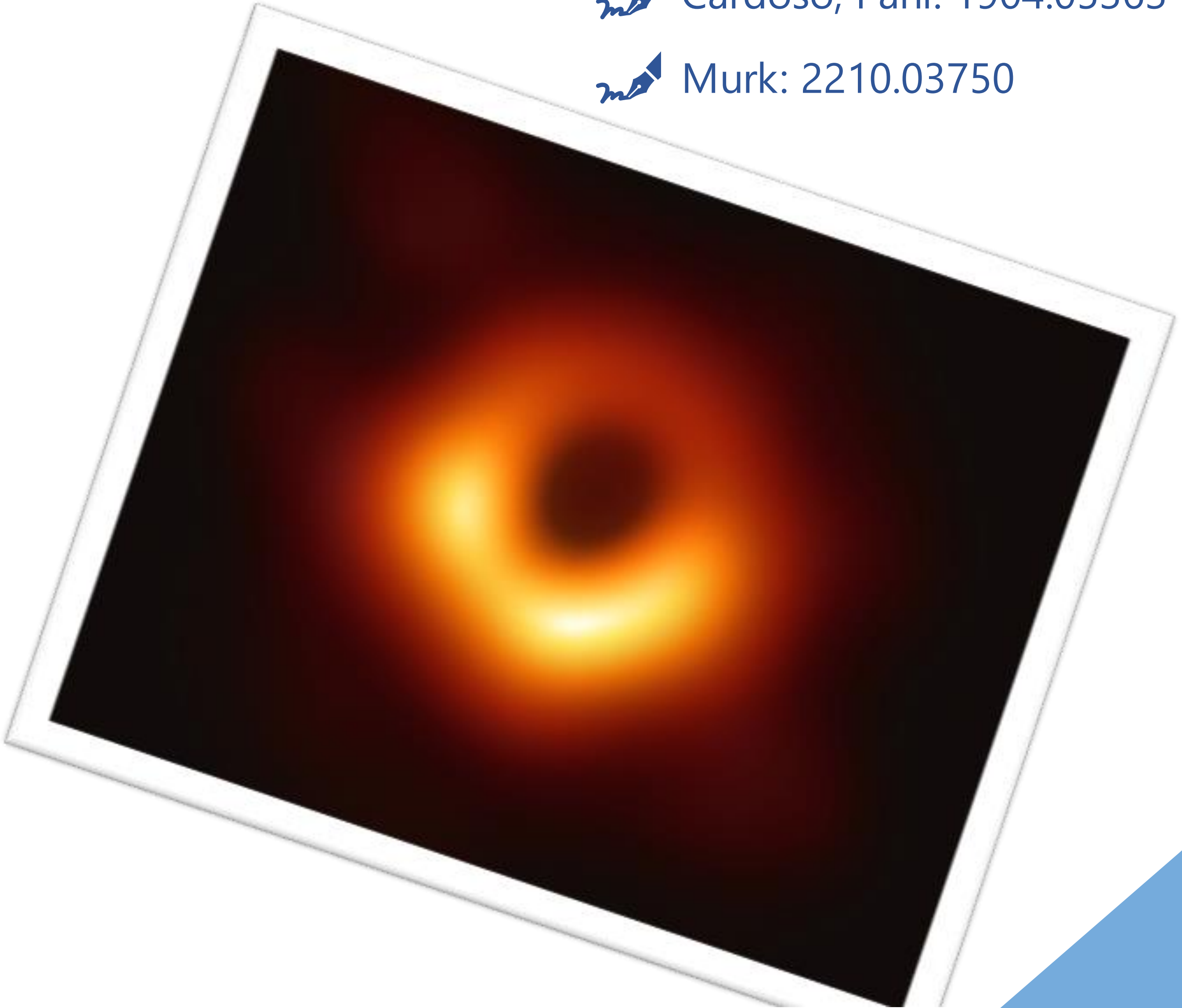

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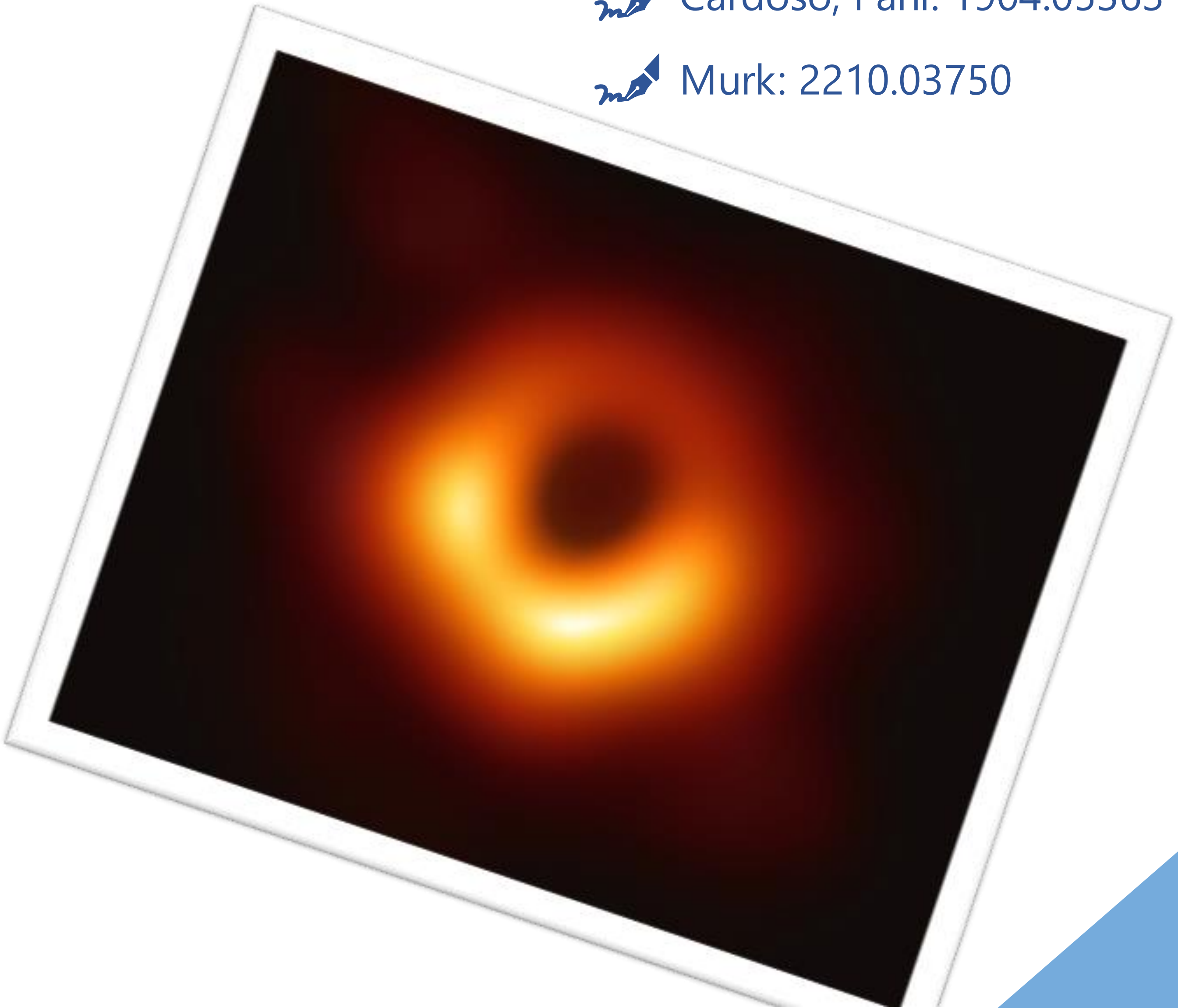
Singularity ✓

Singular black holes



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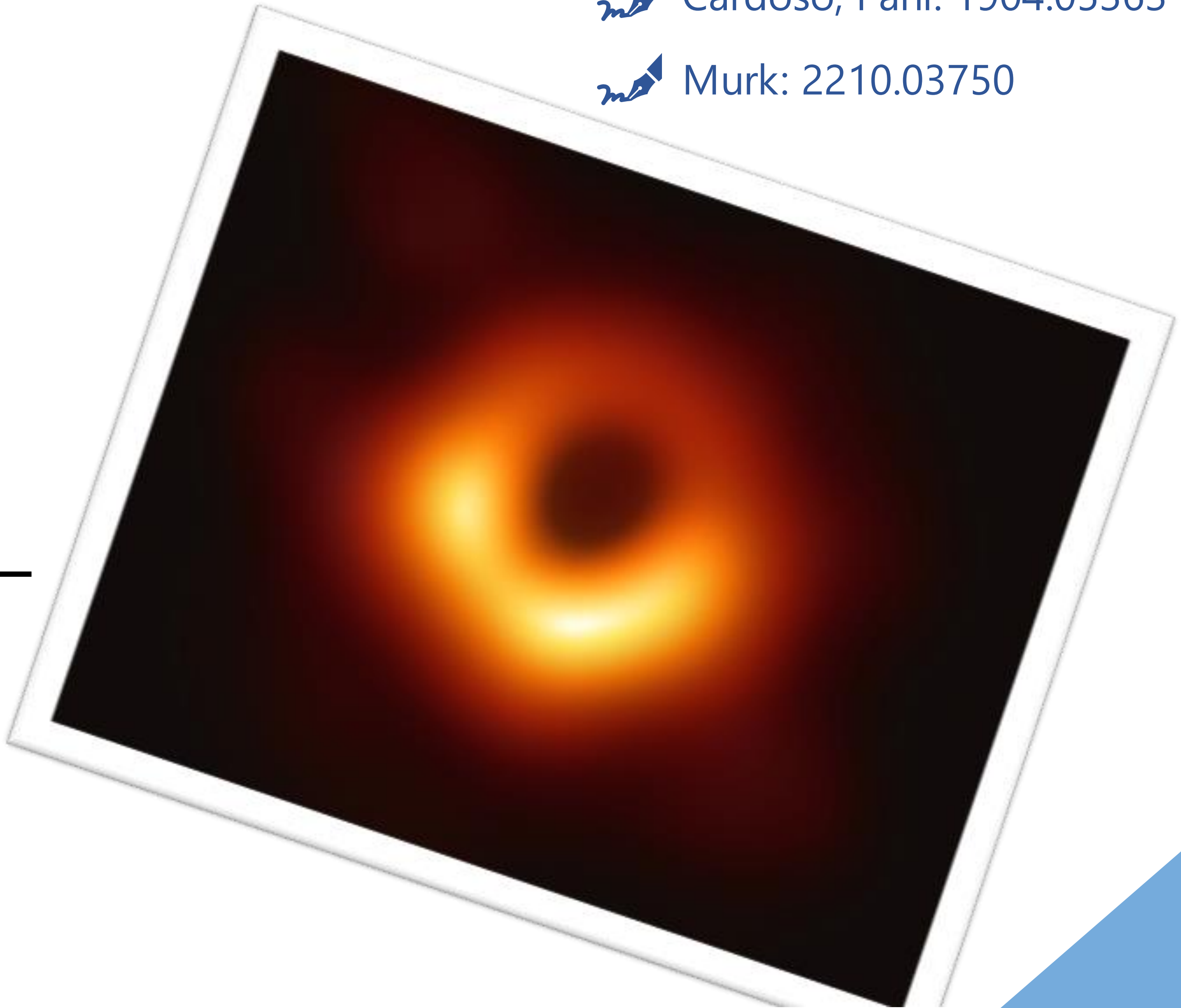
Horizon(s) ✓
Singularity ✗

Regular black holes



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Ultracompact Objects

Compact enough to have a light ring!

True nature?

Horizon(s) ✓ Singularity ✓	Singular black holes
Horizon(s) ✓ Singularity ✗	Regular black holes
Horizon(s) ✗ Singularity ✗	Horizonless configurations

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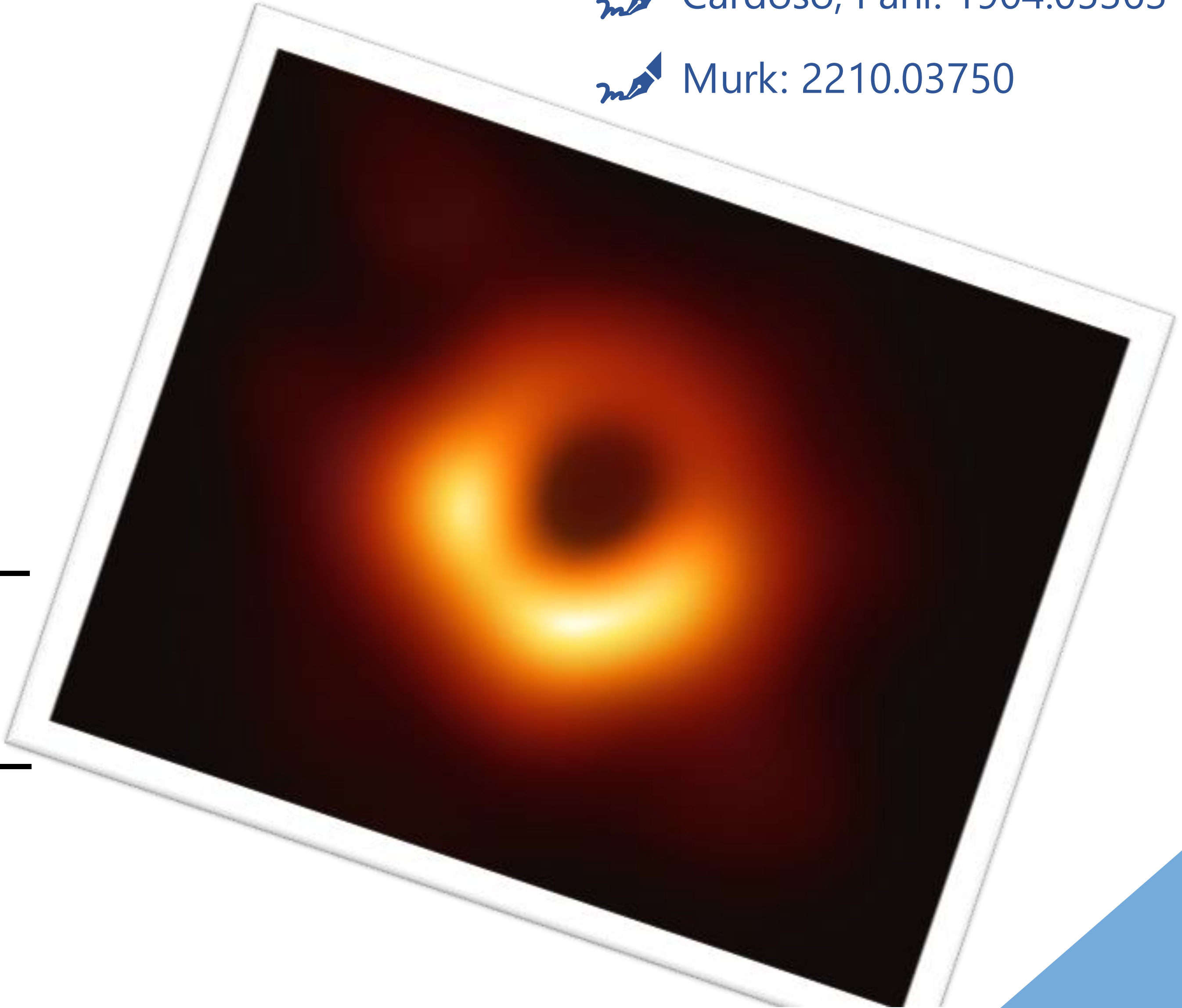

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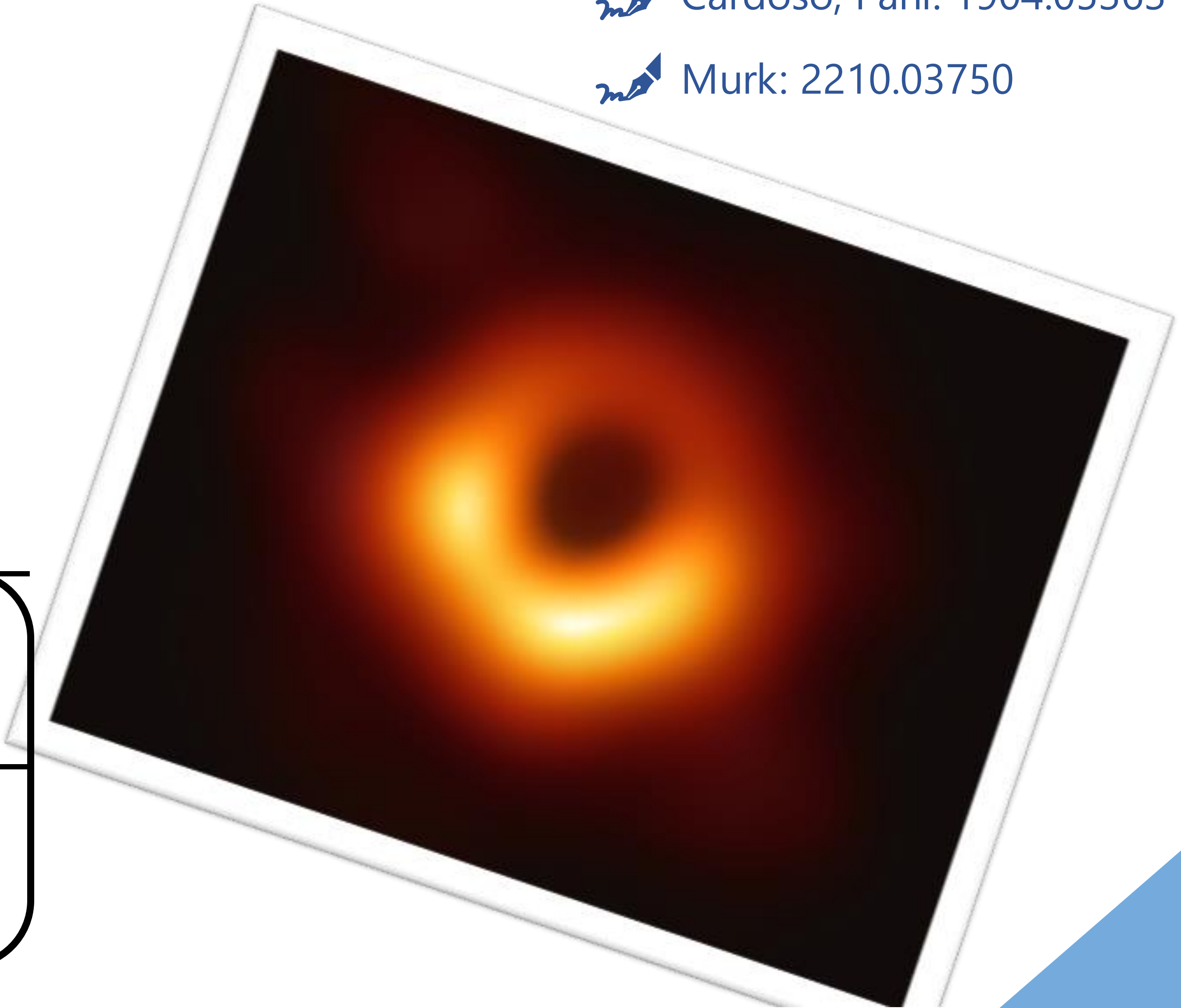

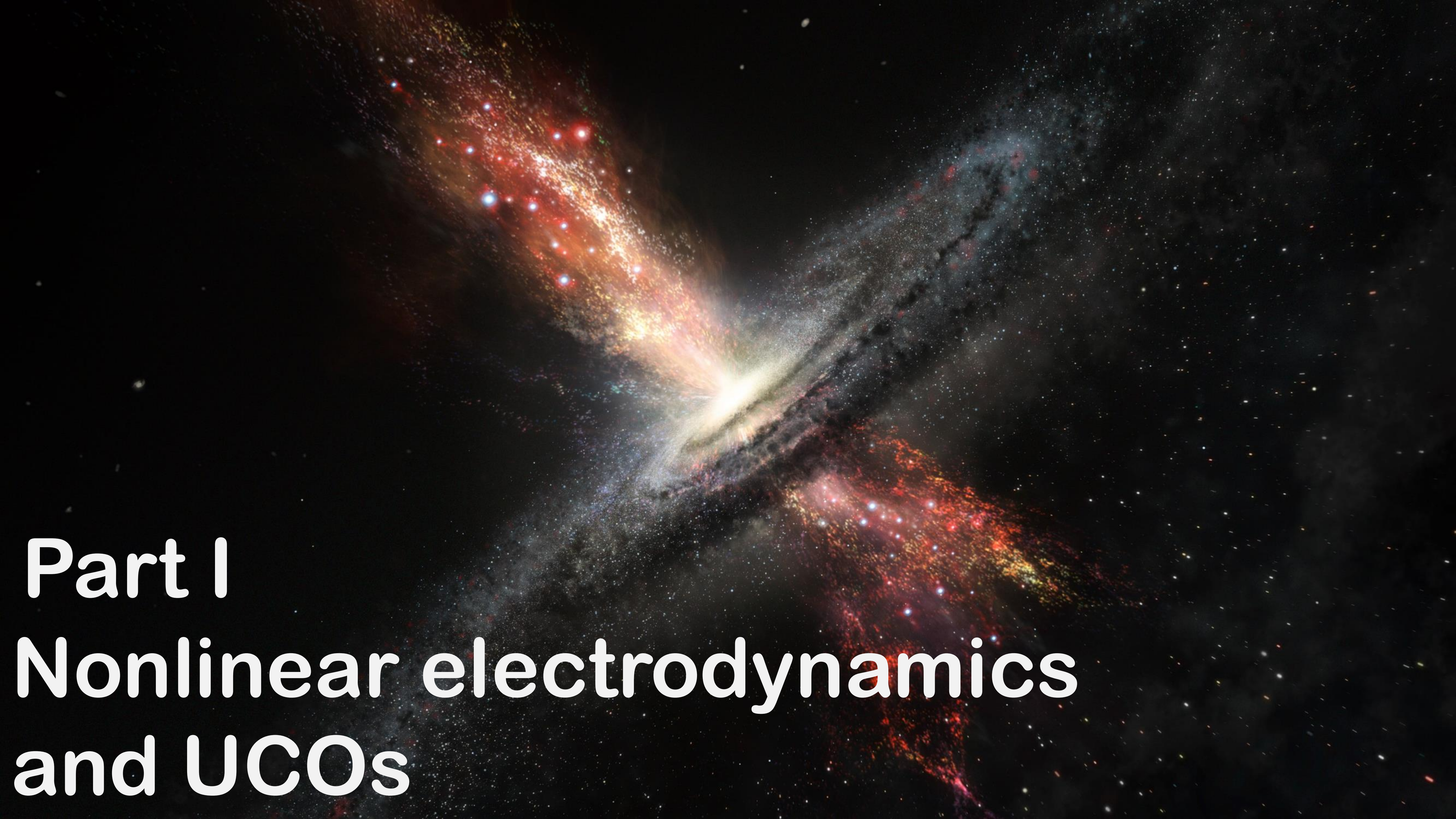

Part I
Nonlinear electrodynamics and UCOs

How do we source them?

- GR + Nonlinear Electrodynamics
- 4D scalar-Einstein-Gauss-Bonnet
- Loop Quantum Gravity

Fan, Wang: 1610.02636

Nojiri, Nashed: 2306.14162

Ashtekar, Olmedo, Singh: 2301.01309

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$$Q_e = 0 \Rightarrow A_\mu = (0, 0, 0, Q_m \cos\theta)$$

Action of the theory

$$I = \frac{1}{16\pi}\int d^4x\sqrt{-g}(R - 2\Lambda - \mathcal{L}(\mathcal{F})), \quad \mathcal{F} = F^{\mu\nu}F_{\mu\nu}, \quad \mathcal{F} = \frac{2Q_m^2}{r^4}$$

NED Lagrangian density

Field Strength

Regular UCOs

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2$$

Regular UCOs $ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2$

Hayward Model

Metric function: $$f_{\mathcal{H}}(r) = 1 - \frac{2mr^2}{r^3 + 2m\ell^2}$$

NED Lagrangian: $$\mathcal{L}(\mathcal{F}) = \frac{12}{\alpha} \frac{(\alpha\mathcal{F})^{3/2}}{(1 + (\alpha\mathcal{F})^{3/4})^2}$$

Weak-field limit: $$\mathcal{L}(\mathcal{F}) \simeq \mathcal{O}(\mathcal{F}^{3/2})$$

Parameters: $$\alpha = 2\ell^2, \quad Q_m = m^{2/3}\left(\frac{\ell}{2}\right)^{1/3}$$

Regular UCOs $ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2$

	Hayward Model	Bardeen Model
Metric function:	$f_{\mathcal{H}}(r) = 1 - \frac{2mr^2}{r^3 + 2m\ell^2}$	$f_{\mathcal{B}}(r) = 1 - \frac{2mr^2}{(r^2 + \ell^2)^{3/2}}$
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Parameters:	$\alpha = 2\ell^2, \quad Q_m = m^{2/3} \left(\frac{\ell}{2}\right)^{1/3}$	$\alpha = \frac{\ell^3}{m}, \quad Q_m = \sqrt{\frac{m\ell}{2}}$

Regular UCOs $ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2$

	Hayward Model	Bardeen Model	Cadoni et al. Model
Metric function:	$f_{\mathcal{H}}(r) = 1 - \frac{2mr^2}{r^3 + 2m\ell^2}$	$f_{\mathcal{B}}(r) = 1 - \frac{2mr^2}{(r^2 + \ell^2)^{3/2}}$	$f_{\mathcal{C}}(r) = 1 - \frac{2mr^2}{(r + \ell)^3}$
NED Lagrangian:	$\mathcal{L}(\mathcal{F}) = \frac{12}{\alpha} \frac{(\alpha\mathcal{F})^{3/2}}{(1 + (\alpha\mathcal{F})^{3/4})^2}$	$\mathcal{L}(\mathcal{F}) = \frac{12}{\alpha} \frac{(\alpha\mathcal{F})^{5/4}}{(1 + \sqrt{\alpha\mathcal{F}})^{5/2}}$	$\mathcal{L}(\mathcal{F}) = \frac{12}{\alpha} \frac{\alpha\mathcal{F}}{(1 + (\alpha\mathcal{F})^{1/4})^4}$
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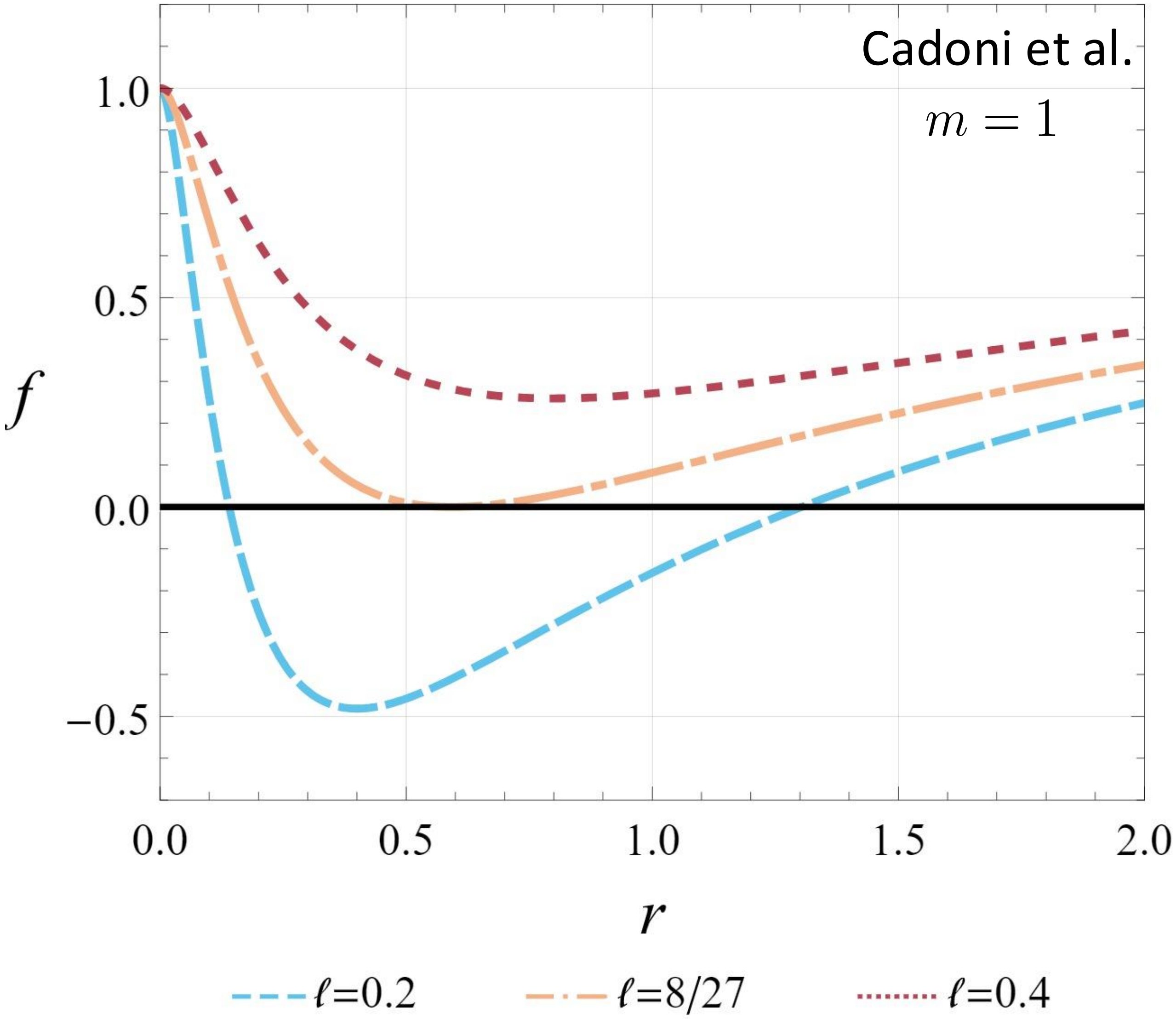
Weak-field limit:	$\mathcal{L}(\mathcal{F}) \simeq \mathcal{O}(\mathcal{F}^{3/2})$	$\mathcal{L}(\mathcal{F}) \simeq \mathcal{O}(\mathcal{F}^{5/4})$	$\mathcal{L}(\mathcal{F}) \simeq \mathcal{O}(\mathcal{F})$
Asymptotics:	$f_{\mathcal{H}}(r) = 1 - \frac{2m}{r} + \frac{4m^2l^2}{r^4} + \mathcal{O}(r^{-7})$	$f_{\mathcal{B}}(r) = 1 - \frac{2m}{r} + \frac{3ml^2}{r^3} + \mathcal{O}(r^{-5})$	$f_{\mathcal{C}}(r) = 1 - \frac{2m}{r} + \frac{6ml}{r^2} + \mathcal{O}(r^{-3})$

Regular UCOs

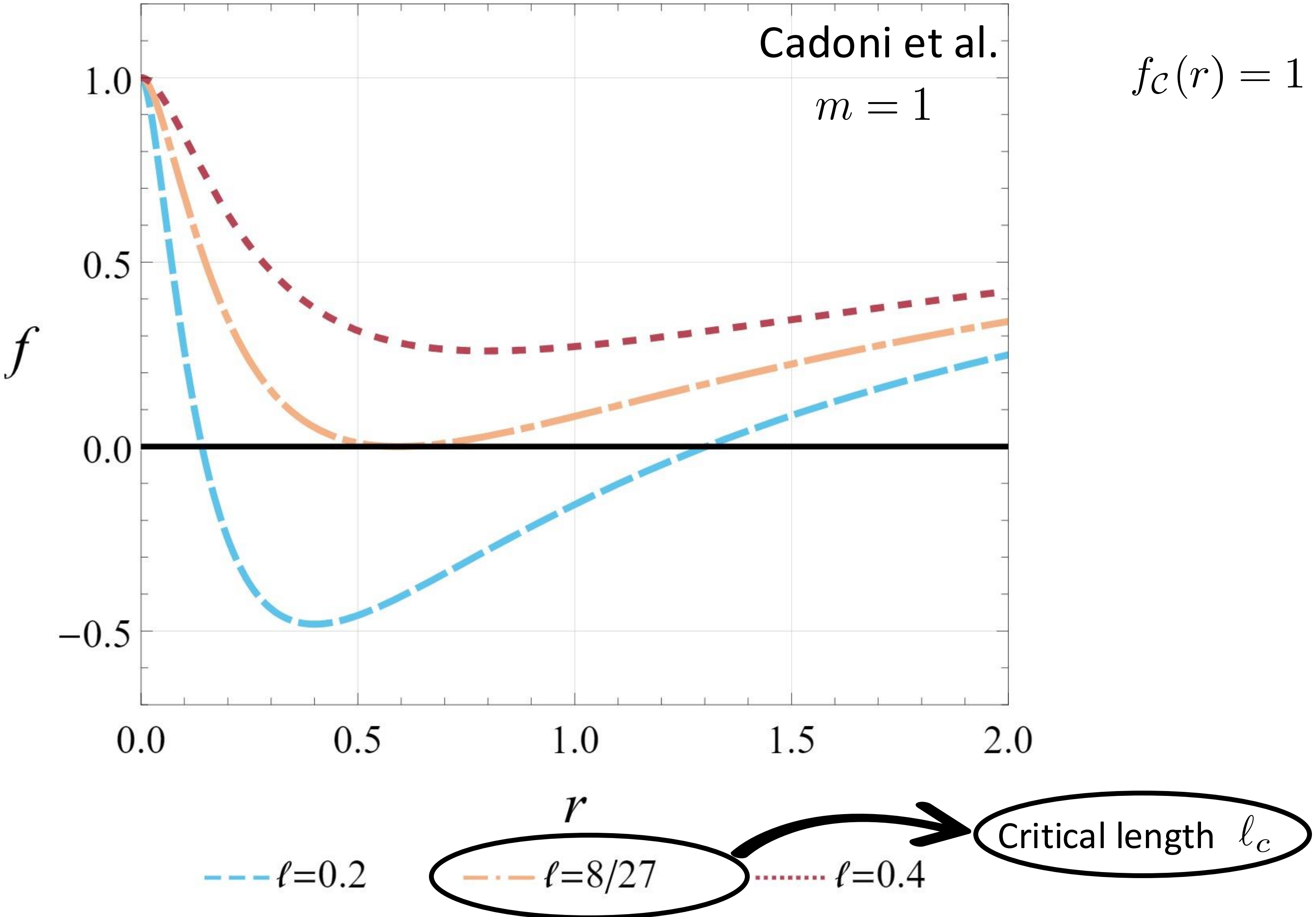
$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2$$



$$f_C(r) = 1 - \frac{2mr^2}{(r+\ell)^3} \qquad \theta_+ = \frac{f_C(r)}{r}$$

Regular UCOs

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2$$



$$f_{\mathcal{C}}(r) = 1 - \frac{2mr^2}{(r+\ell)^3}$$

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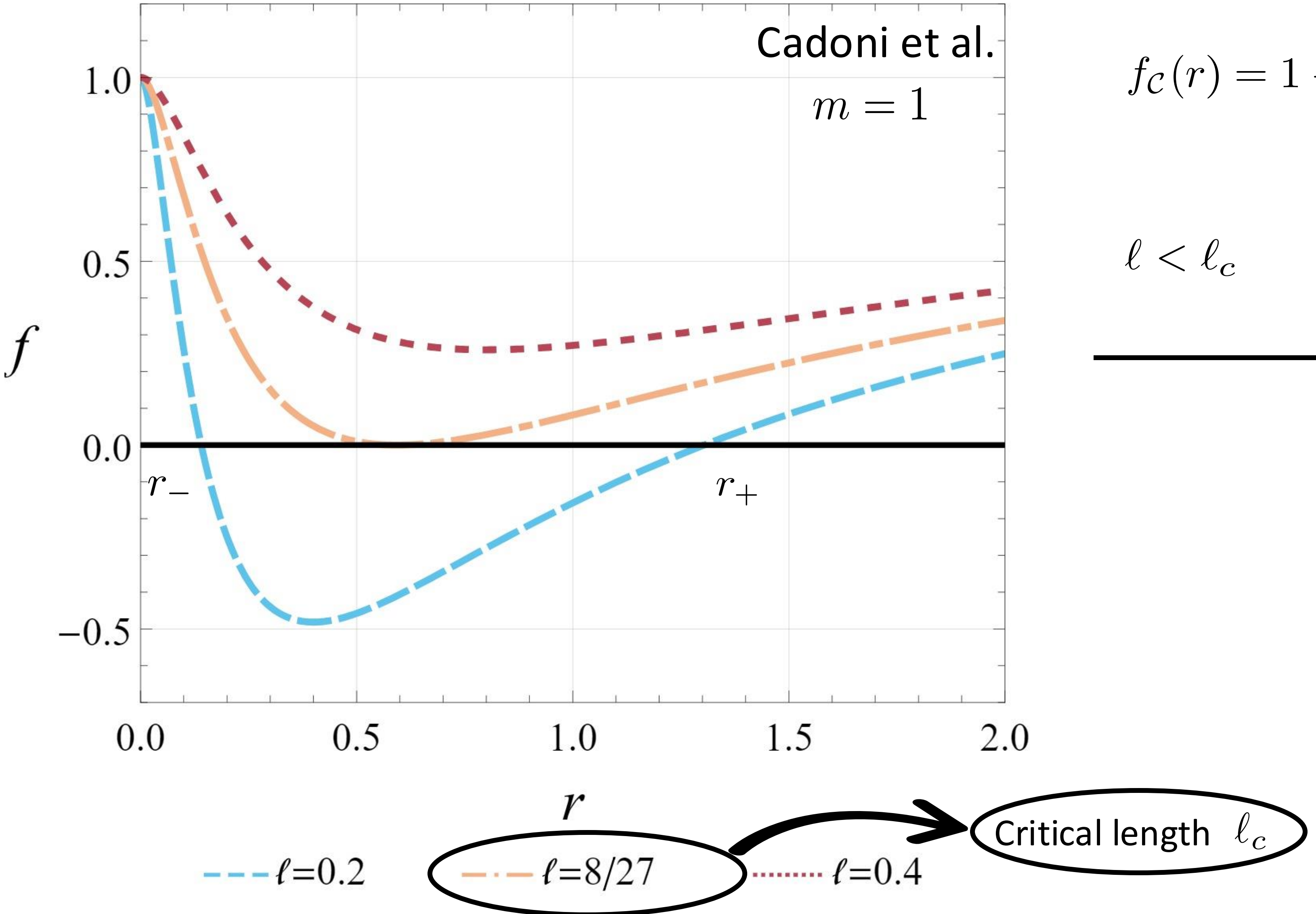
Regular UCOs

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2$$



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$\ell < \ell_c$ — Horizons ✓ Trapped region ✓ — RBH

Regular UCOs

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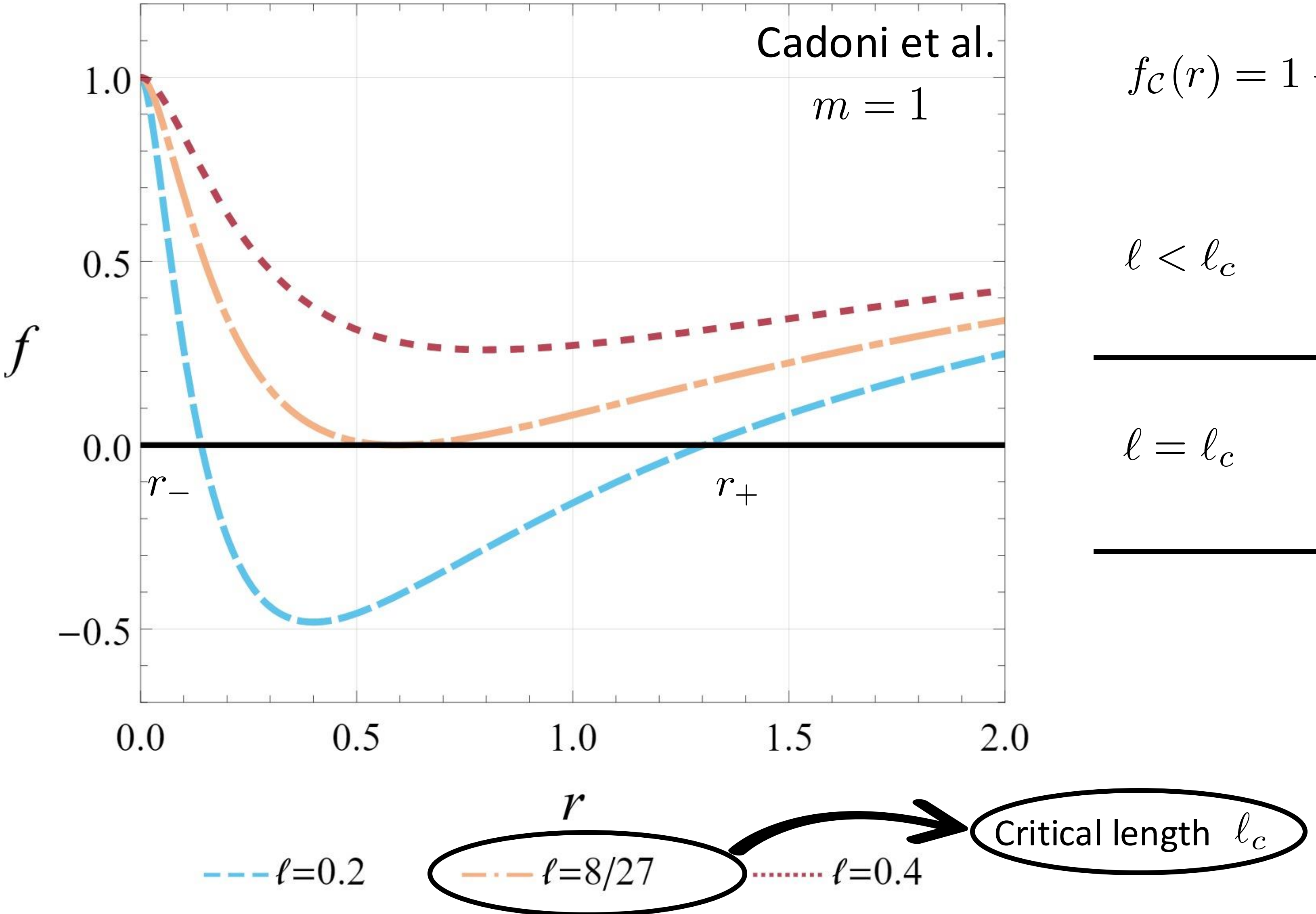
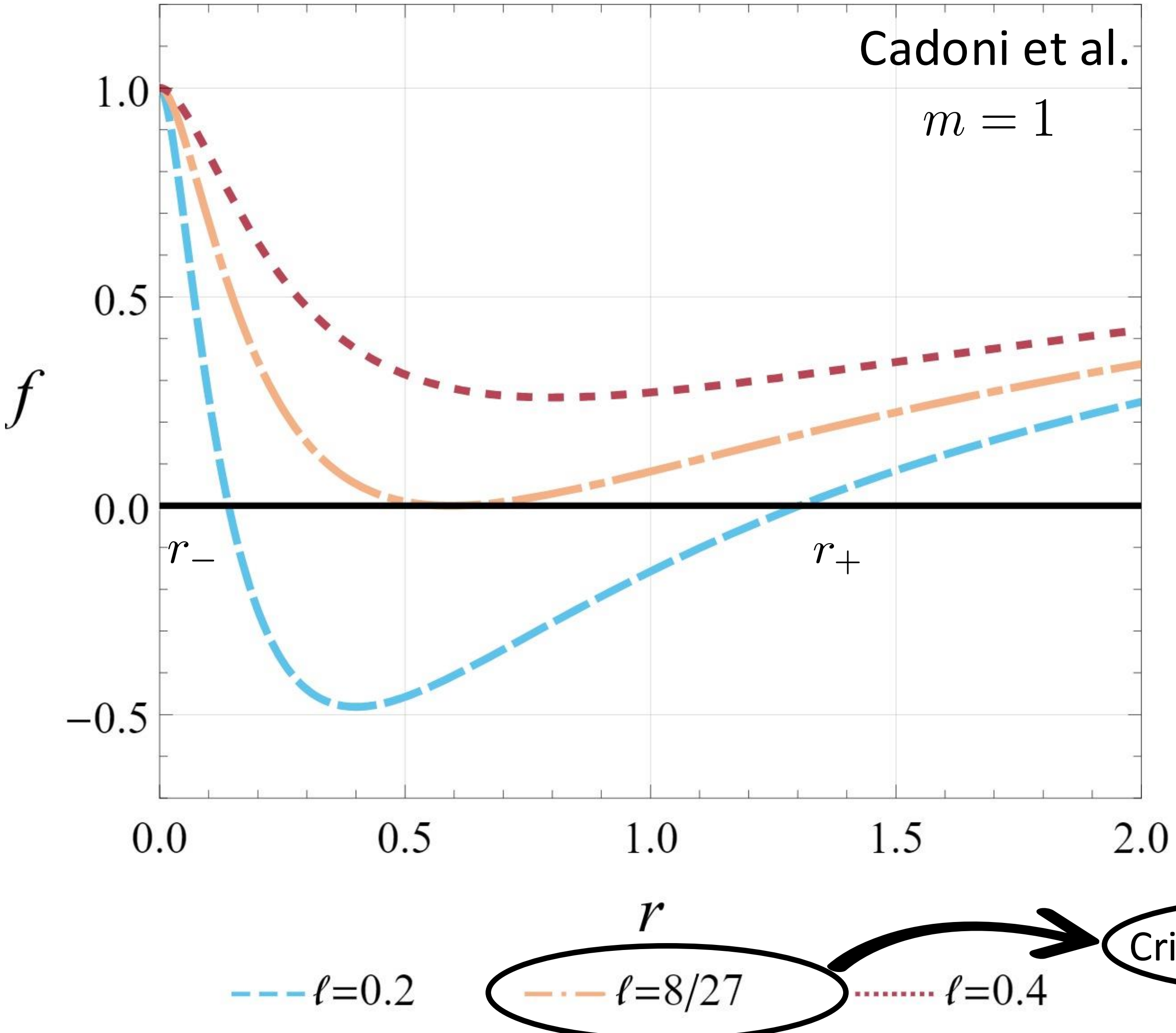


$$f_C(r) = 1 - \frac{2mr^2}{(r+\ell)^3} \qquad \theta_+ = \frac{f_C(r)}{r}$$

$\ell < \ell_c$	Horizons ✓ Trapped region ✓		RBH
$\ell = \ell_c$	Horizon ✓ Trapped region ✗		Extremal RBH

Regular UCOs

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2$$



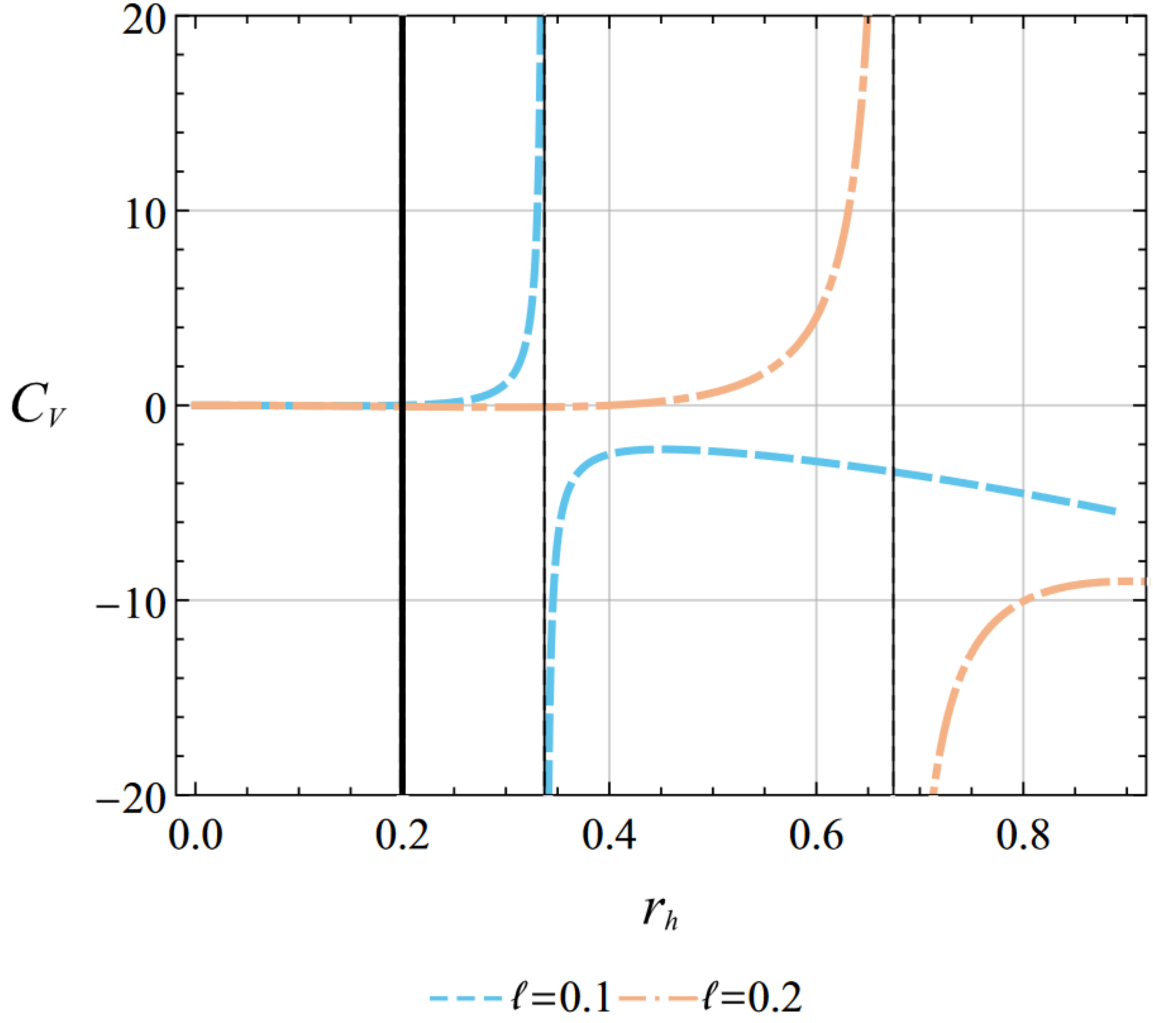
Critical length ℓ_c

$$f_C(r) = 1 - \frac{2mr^2}{(r+\ell)^3} \qquad \theta_+ = \frac{f_C(r)}{r}$$

$\ell < \ell_c$	Horizons ✓ Trapped region ✓	RBH
$\ell = \ell_c$	Horizon ✓ Trapped region ✗	Extremal RBH
$\ell > \ell_c$	Horizon ✗ Trapped region ✗	Horizonless UCO

Minimal Length Constraints

Small Cosmological constant



Thermodynamic quantities in Minkowski

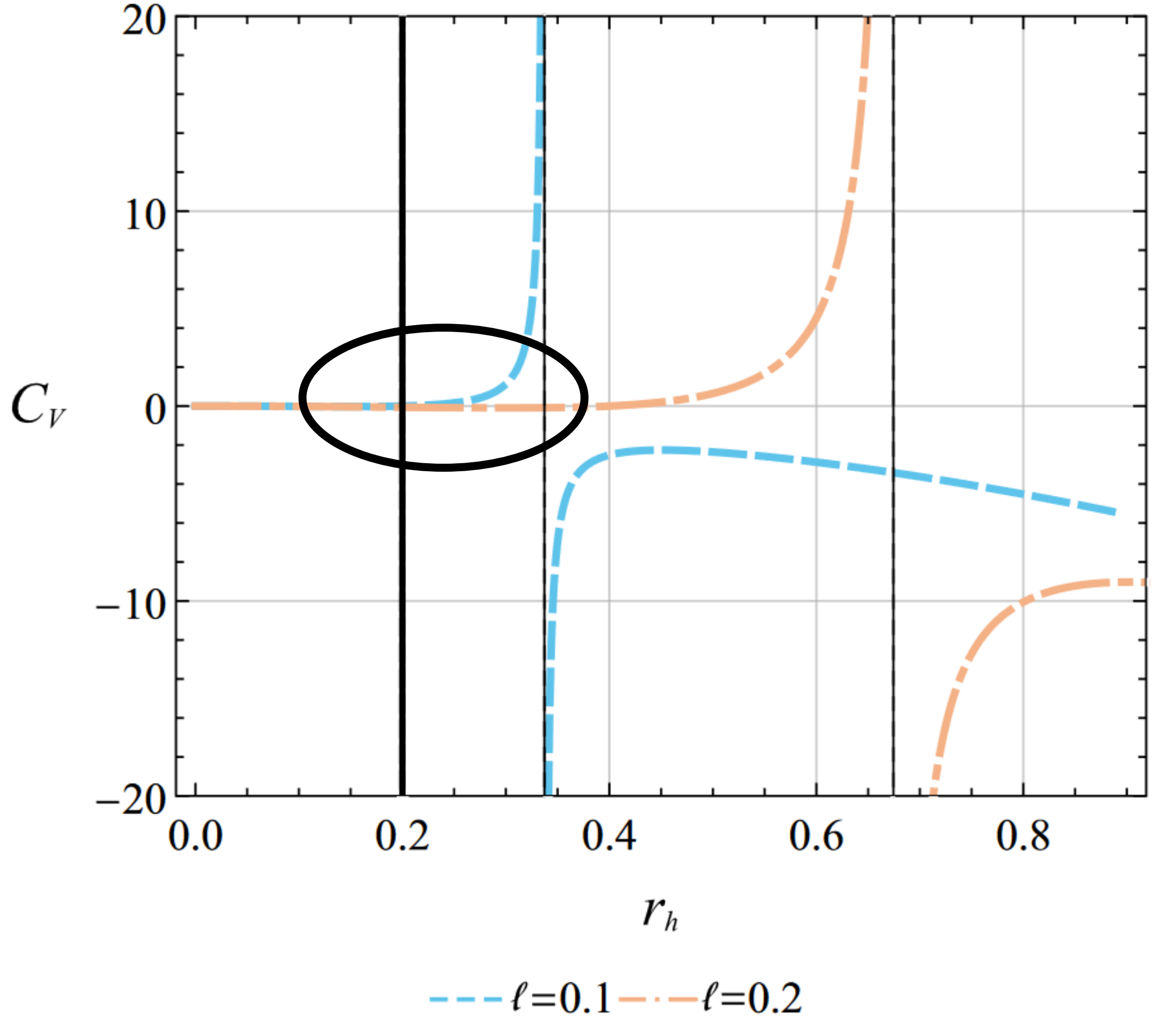
$$0.176 < \frac{\ell}{m} < 0.296$$

Soranidis: 2310.07228

Cadoni et al. : 2211.11585

Minimal Length Constraints

Small Cosmological constant ⇨ Thermodynamic quantities in Minkowski



$$0.176 < \frac{\ell}{m} < 0.296$$

Thermodynamic Stability

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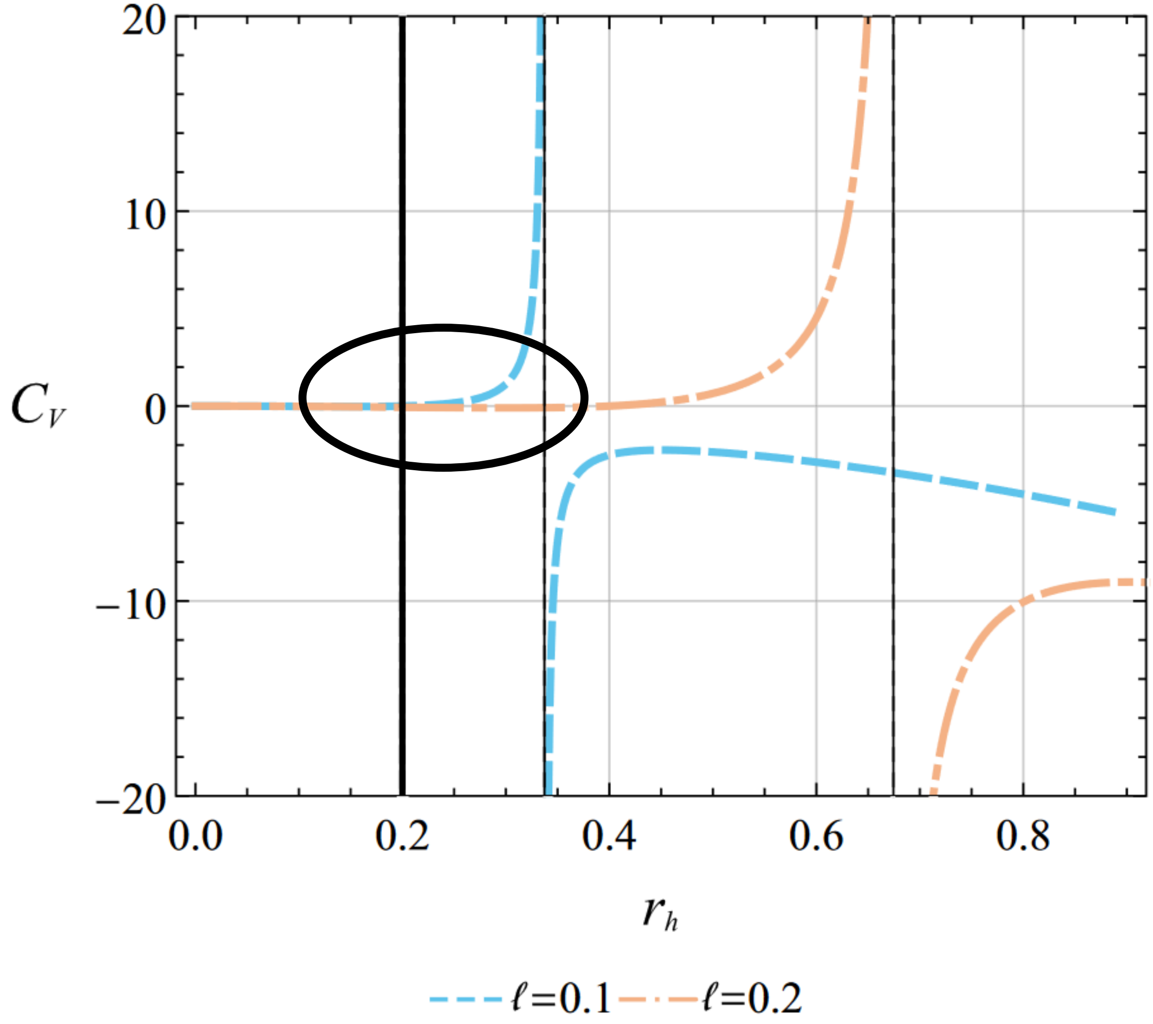
Cadoni et al. : 2211.11585

Minimal Length Constraints

Small Cosmological constant



Thermodynamic quantities in Minkowski

$$0.176 < \frac{\ell}{m} < 0.296$$

Thermodynamic Stability

Extremal Limit

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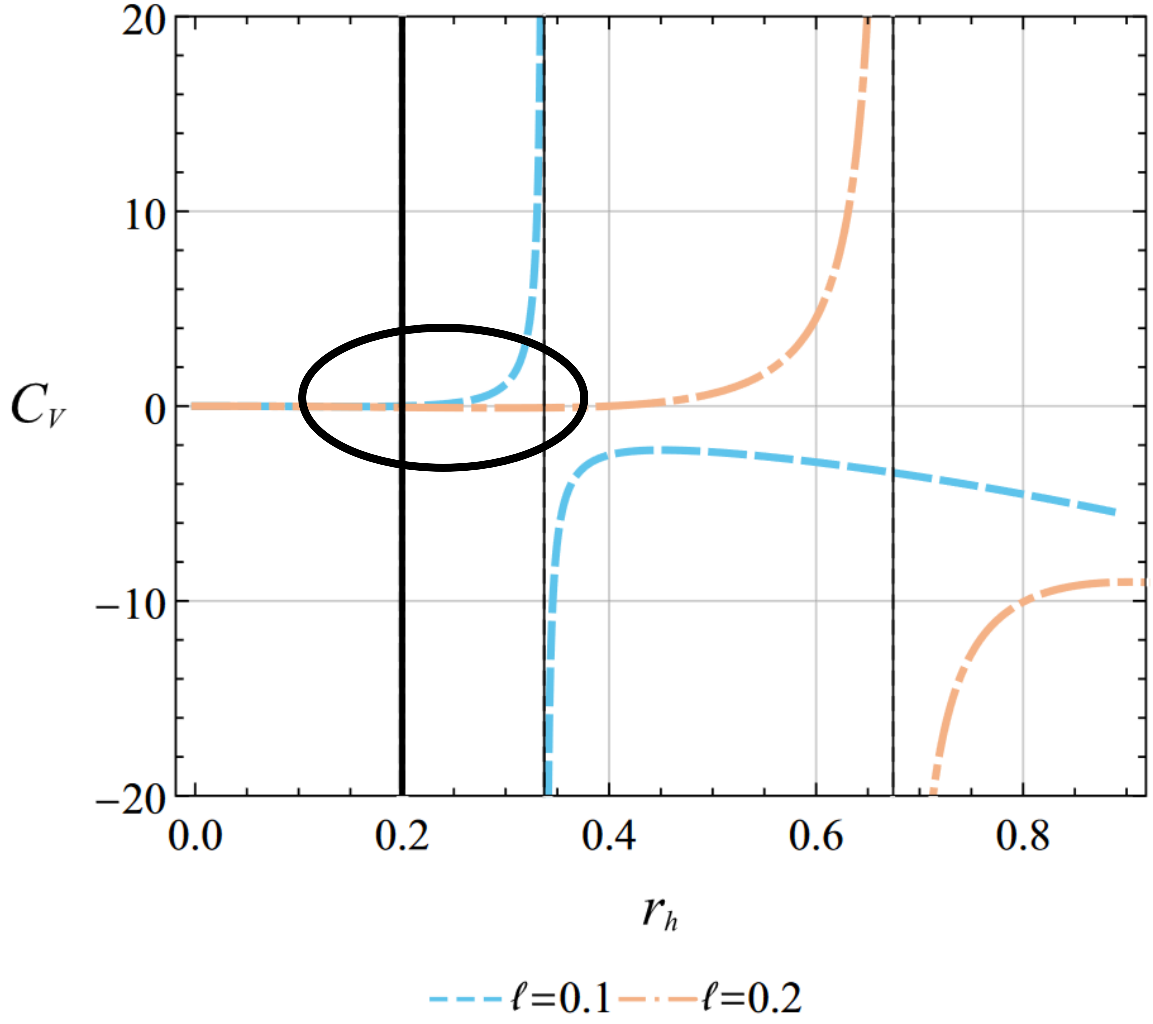
Minimal Length Constraints

Small Cosmological constant



Thermodynamic quantities in Minkowski

$$0.176 < \frac{\ell}{m} < 0.296 < 0.47$$

Thermodynamic Stability

Extremal Limit

Observational Constraints

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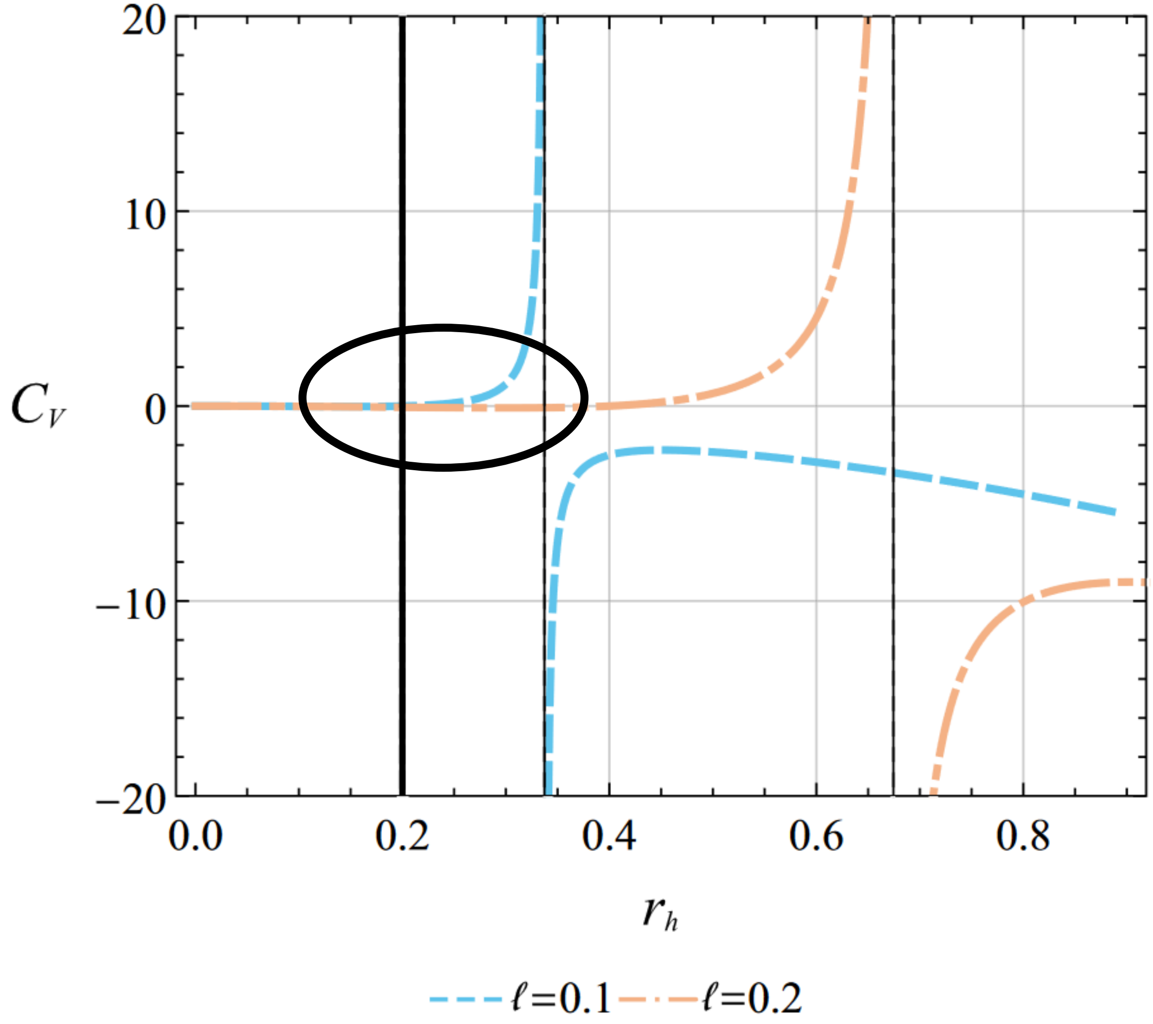
Cadoni et al. : 2211.11585

Minimal Length Constraints

Small Cosmological constant ⇒ Thermodynamic quantities in Minkowski



$$0.176 < \frac{\ell}{m} < 0.296 < 0.47$$

Thermodynamic Stability

Extremal Limit

Observational Constraints

Minimal length scale NOT necessarily the Planck length!

Soranidis: 2310.07228

Cadoni et al. : 2211.11585

Part II
Light rings

Birefringence, light rings, causality

Murk, Soranidis: 2406.07957

The nonlinearity of the theory affects light propagation!

Birefringence, light rings, causality

Murk, Soranidis: 2406.07957

The nonlinearity of the theory affects light propagation!

↓

Unpolarized light ray splits into two light rays!

Birefringence, light rings, causality

Murk, Soranidis: 2406.07957

The nonlinearity of the theory affects light propagation!

Unpolarized light ray splits into two light rays!

Treating light as perturbation!

Background: $g^{\mu\nu}$

Effective: $\bar{g}^{\mu\nu} = g^{\mu\nu} - \frac{4\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}} F^{\mu}{}_{\rho} F^{\rho\nu}$

Birefringence, light rings, causality

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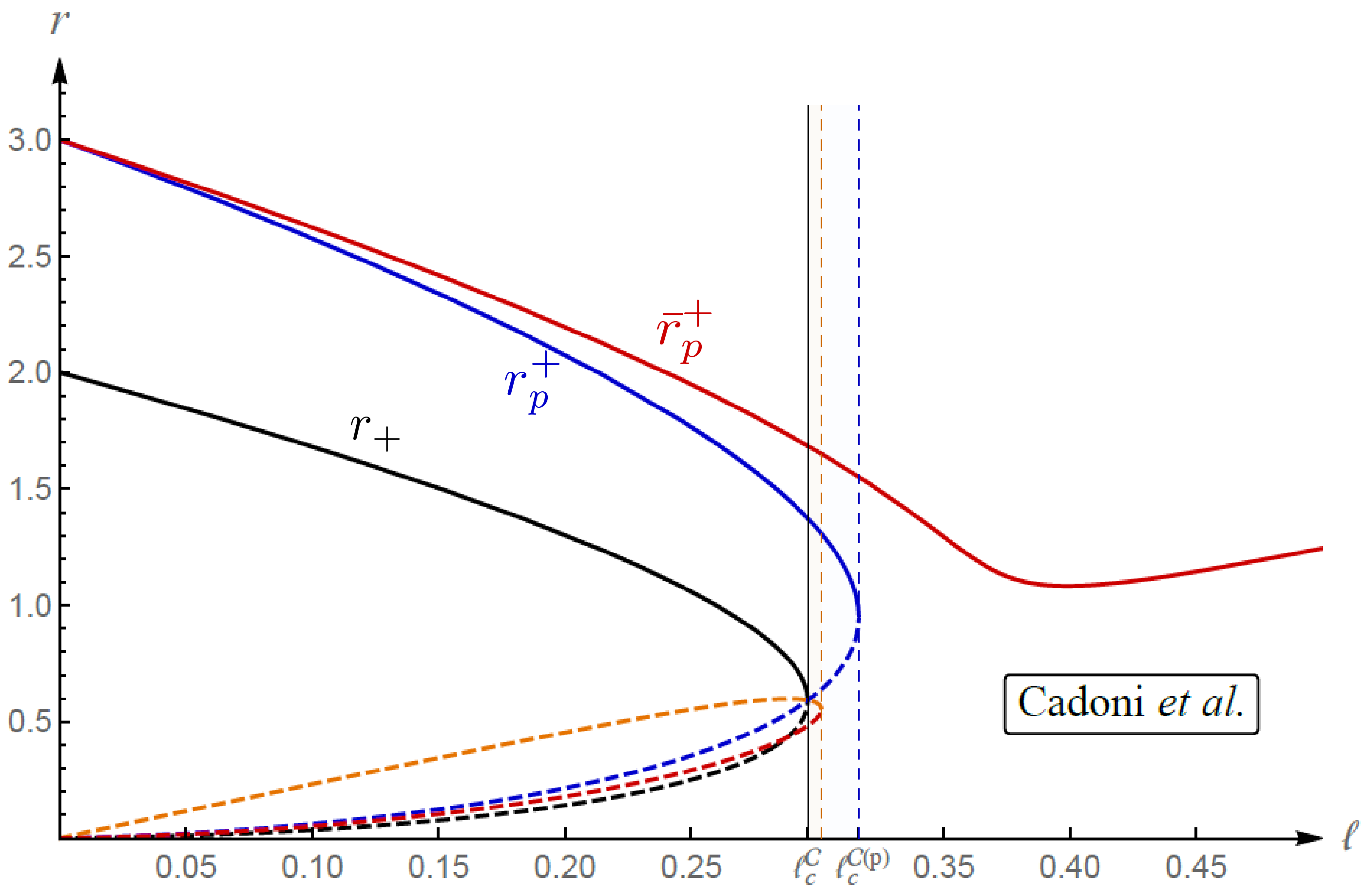

$\ell_c = 0.2963m$

$\bar{\ell}_c^{(p)} = 0.3016m$

$\ell_c^{(p)} = 0.3164m$

Birefringence, light rings, causality

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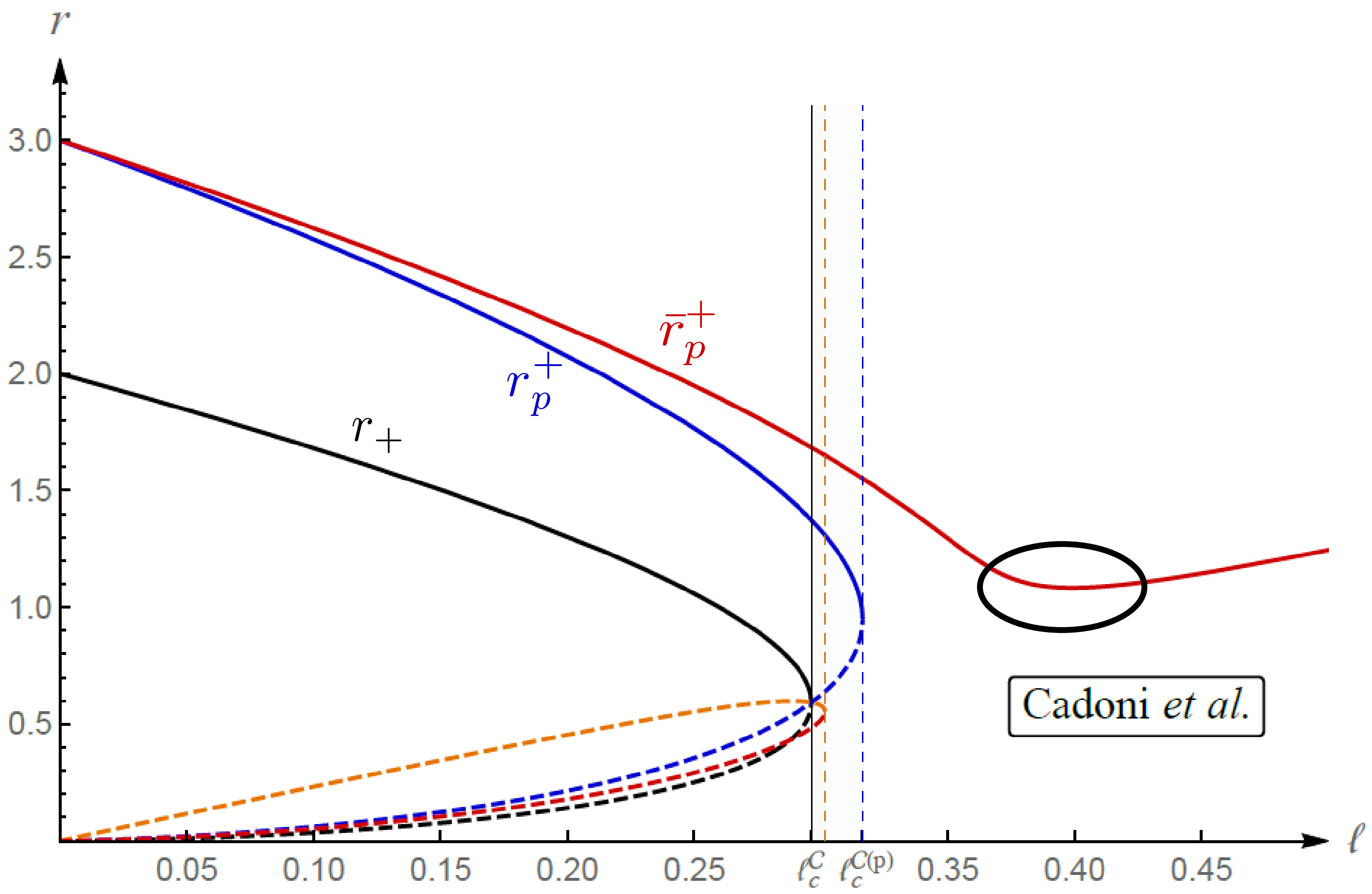


$\ell_c = 0.2963m$

$\bar{\ell}_c^{(p)} = 0.3016m$

$\ell_c^{(p)} = 0.3164m$

$\bar{\ell}^+_{\min} = 0.3991m$

Birefringence, light rings, causality

Murk, Soranidis: 2406.07957

	$0<\ell<\ell_c$	$\ell_c<\ell<\bar{\ell}_c^{(p)}$	$\bar{\ell}_c^{(p)}<\ell<\ell_c^{(p)}$	$\ell>\ell_c^{(p)}$
Nonsingular UCO type	RBH	Horizonless	Horizonless	Horizonless
LRs without birefringence	1	2	2	0
LRs with birefringence	2	5	3	1

Birefringence, light rings, causality

Murk, Soranidis: 2406.07957

	$0<\ell<\ell_c$	$\ell_c<\ell<\overline{\ell}_c^{(p)}$	$\overline{\ell}_c^{(p)}<\ell<\ell_c^{(p)}$	$\ell>\ell_c^{(p)}$
Nonsingular UCO type	RBH	Horizonless	Horizonless	Horizonless
LRs without birefringence	1	2	2	0
LRs with birefringence	2	5	3	1

Birefringence, light rings, causality

Murk, Soranidis: 2406.07957

	$0 < \ell < \ell_c$	$\ell_c < \ell < \bar{\ell}_c^{(p)}$	$\bar{\ell}_c^{(p)} < \ell < \ell_c^{(p)}$	$\ell > \ell_c^{(p)}$
Nonsingular UCO type	RBH	Horizonless	Horizonless	Horizonless
LRs without birefringence	1	2	2	0
LRs with birefringence	2	5	3	1

Knowledge of underlying theory is necessary!

Birefringence, light rings, causality

Murk, Soranidis: 2406.07957

$$k_\mu = \left(-\omega, \sqrt{g_{11}}|\vec{k}|\cos\eta, 0, \sqrt{g_{33}}|\vec{k}|\sin\eta\right) \longrightarrow$$ Propagation vector

$$|\vec{k}|^2 = g^{ij}k_ik_j, \quad i,j = 1,2,3$$

Birefringence, light rings, causality

Murk, Soranidis: 2406.07957

$$k_\mu = \left(-\omega, \sqrt{g_{11}}|\vec{k}|\cos\eta, 0, \sqrt{g_{33}}|\vec{k}|\sin\eta\right)$$ → Propagation vector

$$|\vec{k}|^2 = g^{ij}k_i k_j, \quad i,j = 1,2,3$$

Background Geometry

$$g^{\mu\nu}k_\mu k_\nu = 0$$

$$v_{\text{ph}} = \frac{\omega}{|\vec{k}|} = \sqrt{f(r)}$$

Birefringence, light rings, causality

Murk, Soranidis: 2406.07957

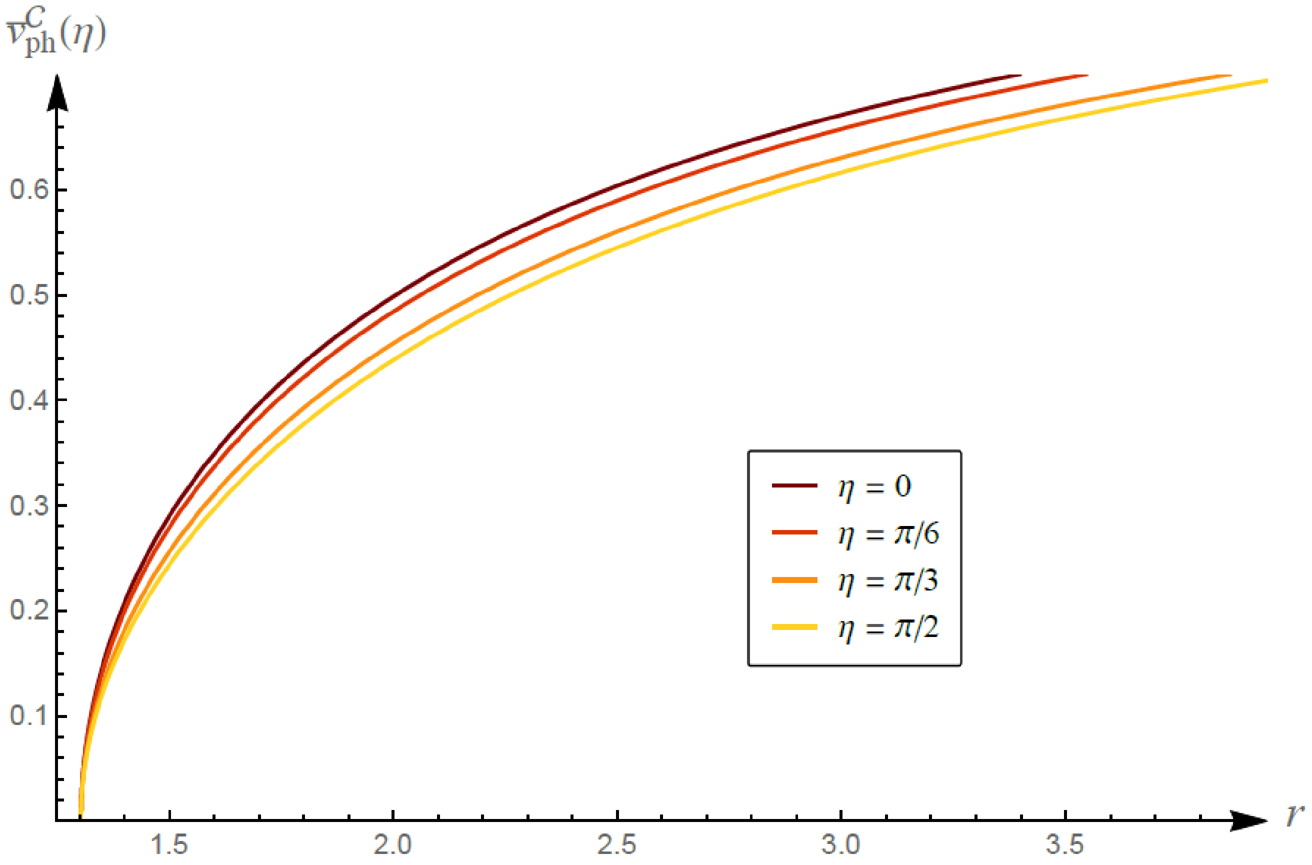
$$k_\mu = \left(-\omega, \sqrt{g_{11}}|\vec{k}|\cos\eta, 0, \sqrt{g_{33}}|\vec{k}|\sin\eta\right)$$ → Propagation vector

$$|\vec{k}|^2 = g^{ij}k_i k_j, \quad i,j = 1,2,3$$

Background Geometry	Effective Geometry
$g^{\mu\nu}k_\mu k_\nu = 0$	$\bar{g}^{\mu\nu}k_\mu k_\nu = 0$
$v_{\text{ph}} = \frac{\omega}{\lvert\vec{k}\rvert} = \sqrt{f(r)}$	$\bar{v}_{\text{ph}}(\eta) = \frac{\omega}{\lvert\vec{k}\rvert} = \sqrt{f(r)\left(1 + \frac{2\mathcal{F}\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}}\sin^2\eta\right)}$

Birefringence, light rings, causality

Murk, Soranidis: 2406.07957



$$f_{\mathcal{C}}(r) = 1 - \frac{2mr^2}{(r+l)^3}$$

$$v^{(\mathcal{C})}_{\rm ph} = \sqrt{f_{\mathcal{C}}(r)}$$

$$\bar{v}^{(\mathcal{C})}_{\rm ph}(\eta) = \sqrt{f_{\mathcal{C}}(r)\left(1 - \frac{5\ell}{2(r+\ell)}\sin^2\eta\right)}$$

Birefringence, light rings, causality

Murk, Soranidis: 2406.07957

Hayward Model

$$f_{\mathcal{H}}(r) = 1 - \frac{2mr^2}{r^3 + 2m\ell^2}$$

$$v_{\rm ph}^{(\mathcal{H})} = \sqrt{f_{\mathcal{H}}(r)}$$

$$\bar{v}_{\rm ph}^{(\mathcal{H})}(\eta) = \sqrt{f_{\mathcal{H}}(r)\left(1 + \frac{r^3 - 7m\ell^2}{r^3 + 2m\ell^2}\sin^2\eta\right)}$$

Causal ✗

Bardeen Model

$$f_{\mathcal{B}}(r) = 1 - \frac{2mr^2}{(r^2 + \ell^2)^{3/2}}$$

$$v_{\rm ph}^{(\mathcal{B})} = \sqrt{f_{\mathcal{B}}(r)}$$

$$\bar{v}_{\rm ph}^{(\mathcal{B})}(\eta) = \sqrt{f_{\mathcal{B}}(r)\left(1 + \frac{r^2 - 6\ell^2}{2(r^2 + \ell^2)}\sin^2\eta\right)}$$

Causal ✗

Cadoni et al. Model

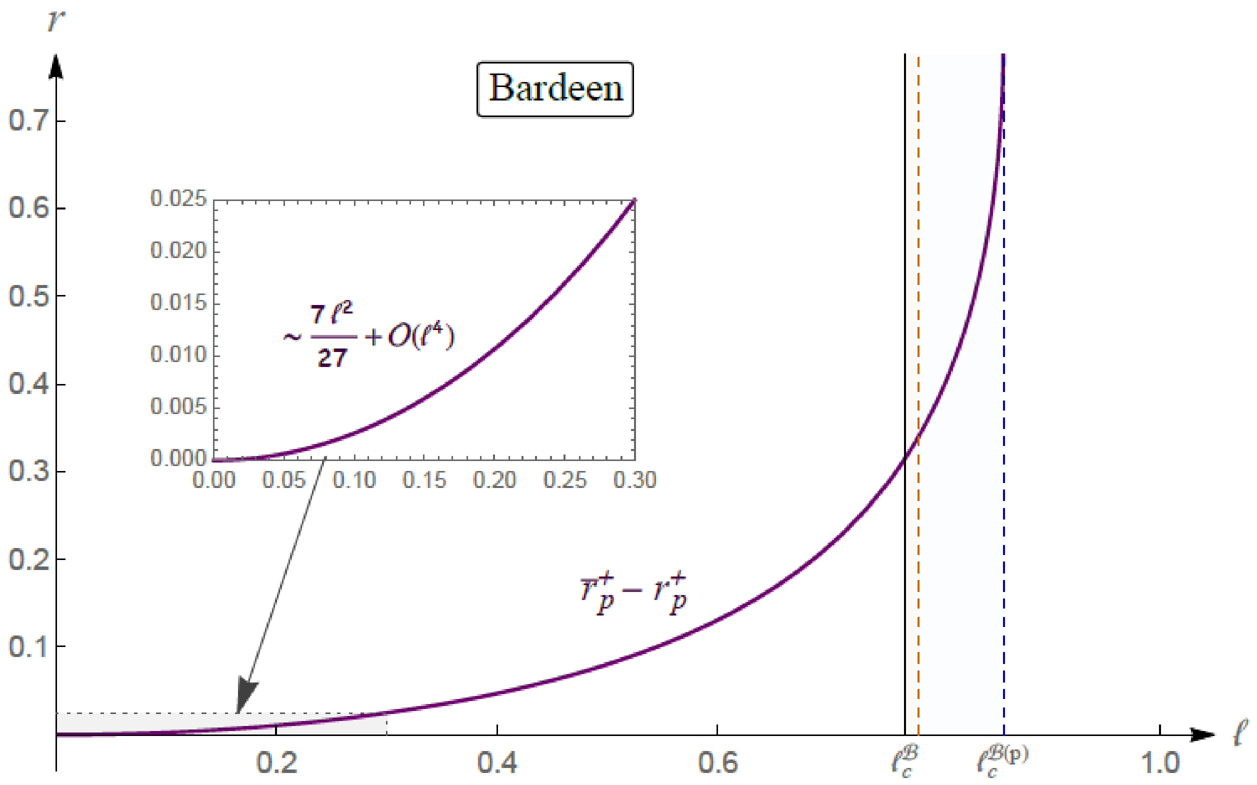
$$f_{\mathcal{C}}(r) = 1 - \frac{2mr^2}{(r + l)^3}$$

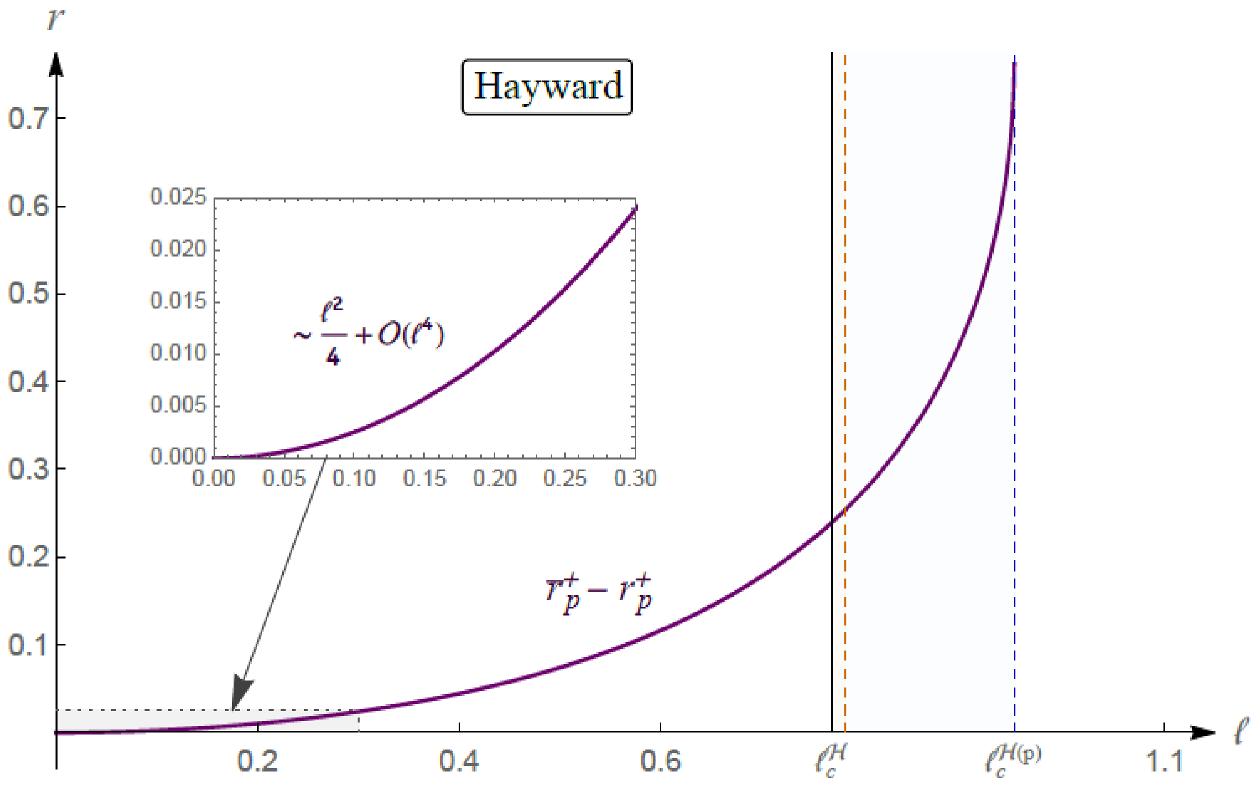
$$v_{\rm ph}^{(\mathcal{C})} = \sqrt{f_{\mathcal{C}}(r)}$$

$$\bar{v}_{\rm ph}^{(\mathcal{C})}(\eta) = \sqrt{f_{\mathcal{C}}(r)\left(1 - \frac{5\ell}{2(r + \ell)}\sin^2\eta\right)}$$

Causal ✓

Birefringence, light rings, causality





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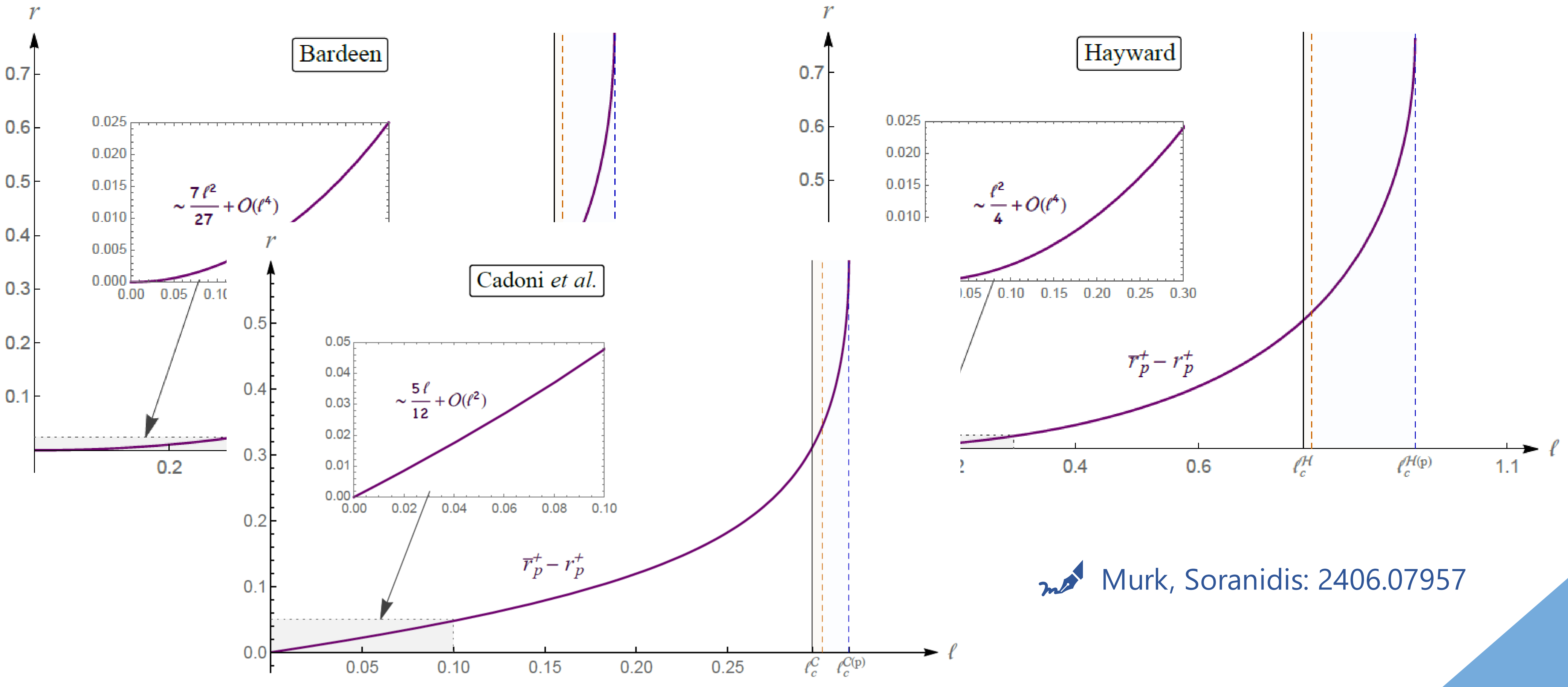
Birefringence, light rings, causality



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Part III
Conclusions

Thoughts…

- What is a good effective description in 4D of such objects in the absence of a full quantum gravity theory?
- What are the ultracompact objects we see?
- Can we distinguish between them through light rings without additional information?
- Future ambition: Can we see such observational signatures in the near future?

OIST

CSIRO

Summary of Results

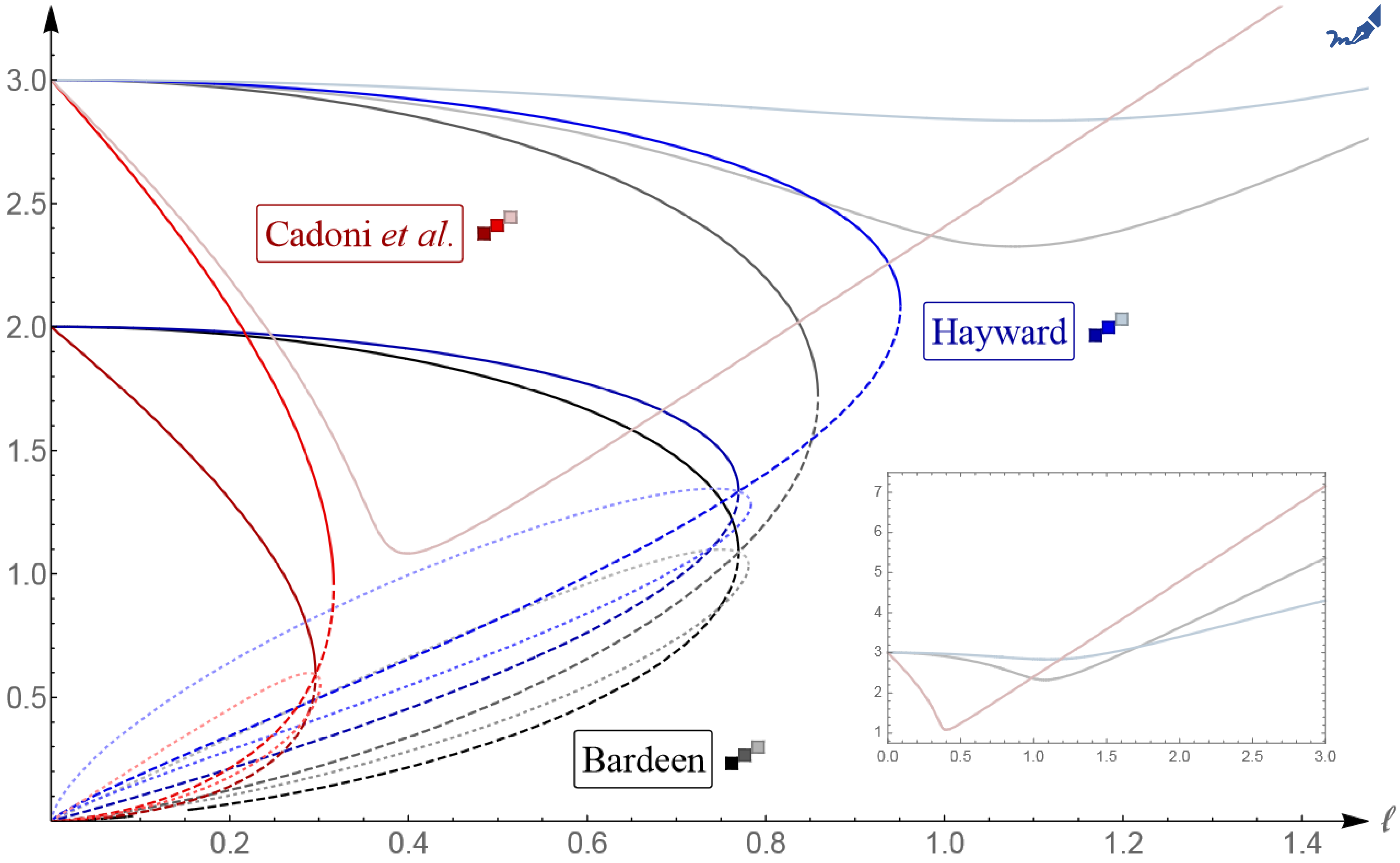
- Birefringence and light ring splitting for NED-sourced UCOs
- Cases of horizonful and horizonless objects with same number of light rings
- RBHs with the Maxwell weak-field limit have larger separation of light rings
- Horizonless UCOs with one unstable light ring

Thank you!

Back-up Slides

Birefringence, light rings, causality

Murk, Soranidis: 2406.07957



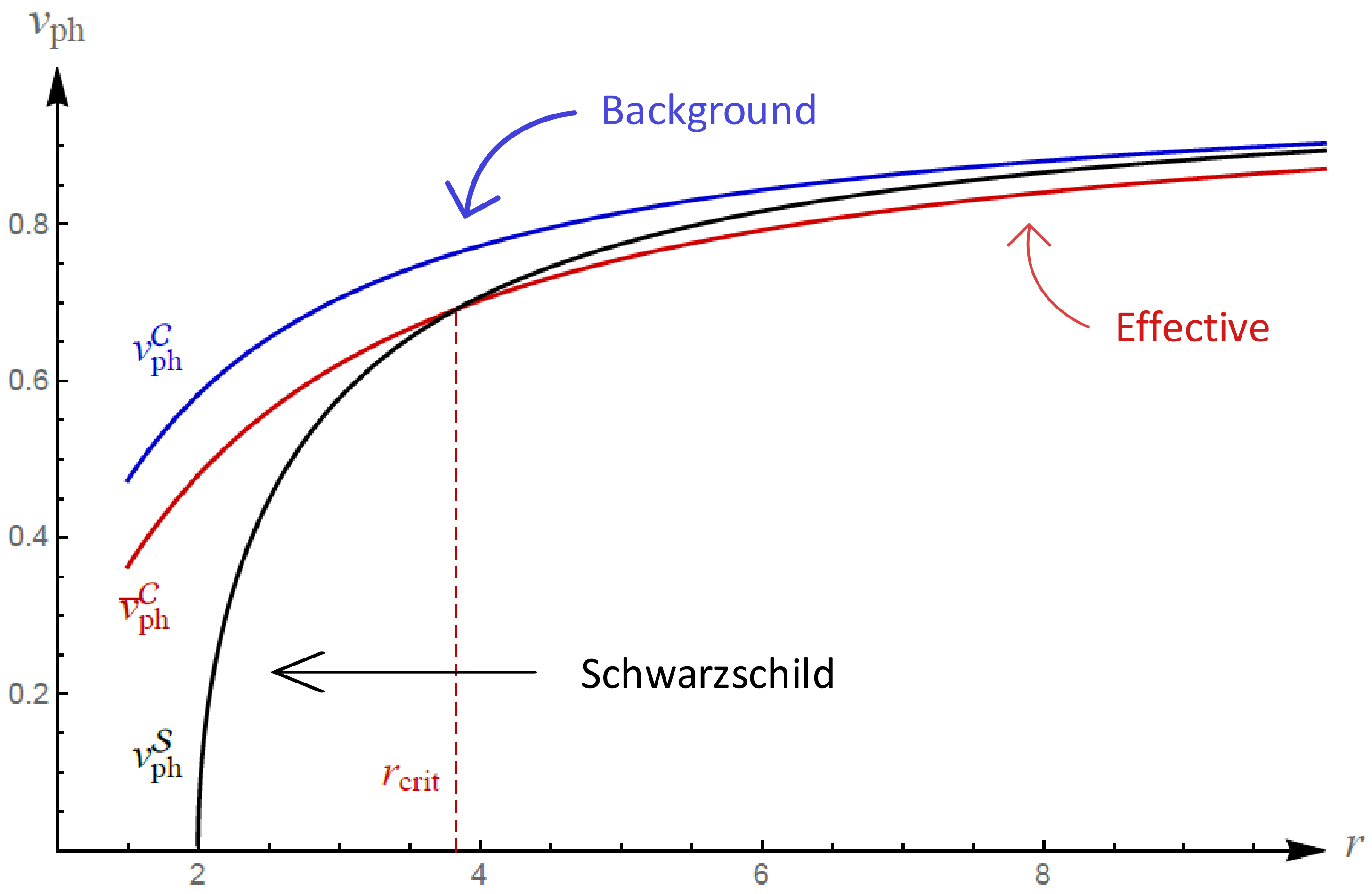
Birefringence, light rings, causality

Murk, Soranidis: 2406.07957

RBH model	ℓ_c	$\bar{\ell}_c^{(p)}$	$\ell_c^{(p)}$	$\bar{\ell}^{+}_{\text{min}}$
Bardeen	$0.7698M$	$0.7811M$	$0.8587M$	$1.0766M$
Hayward	$0.7698M$	$0.7836M$	$0.9509M$	$1.1004M$
Cadoni *et al.*	$0.2963M$	$0.3016M$	$0.3164M$	$0.3991M$

Birefringence, light rings, causality

Murk, Soranidis: 2406.07957



$$\eta = \frac{\pi}{2}$$

↓

Circular trajectories!