30 Years of the ASGRG
1994-2024

Cosmology's Years in the Wilderness

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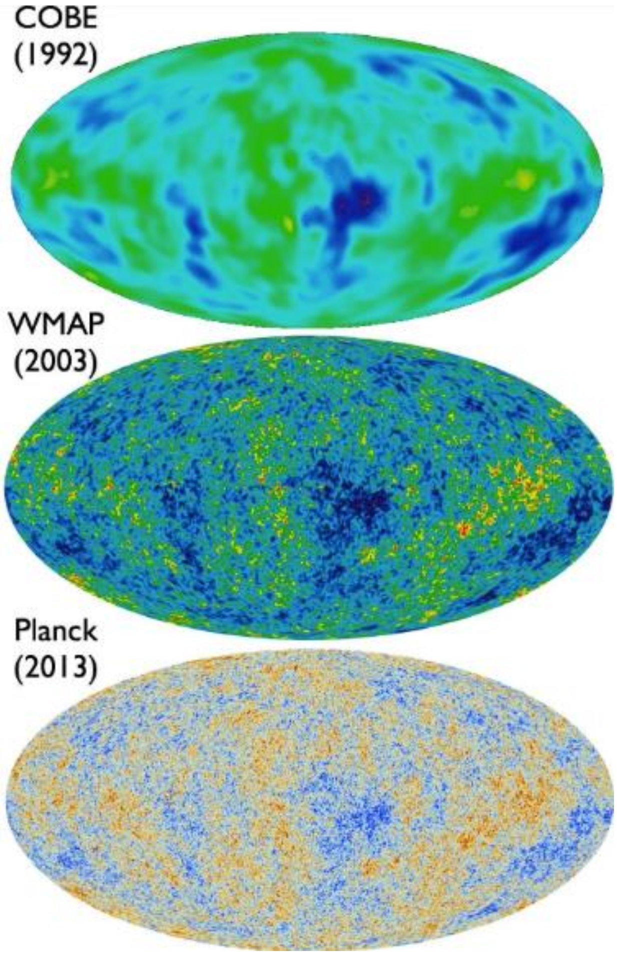
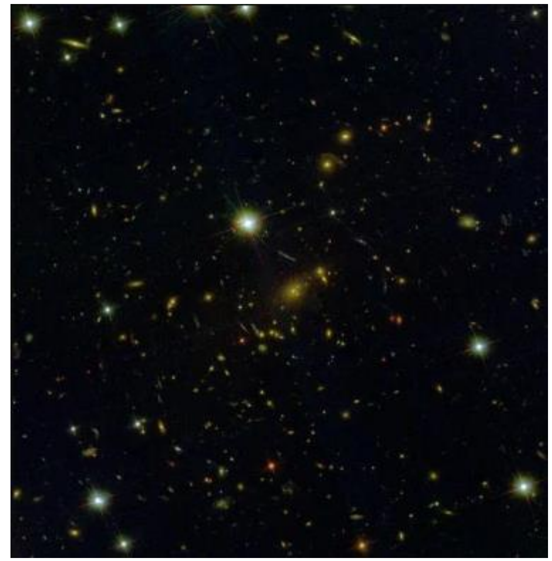
1989 to Today

A Golden Age for Observational Cosmology



Cosmic Microwave Background



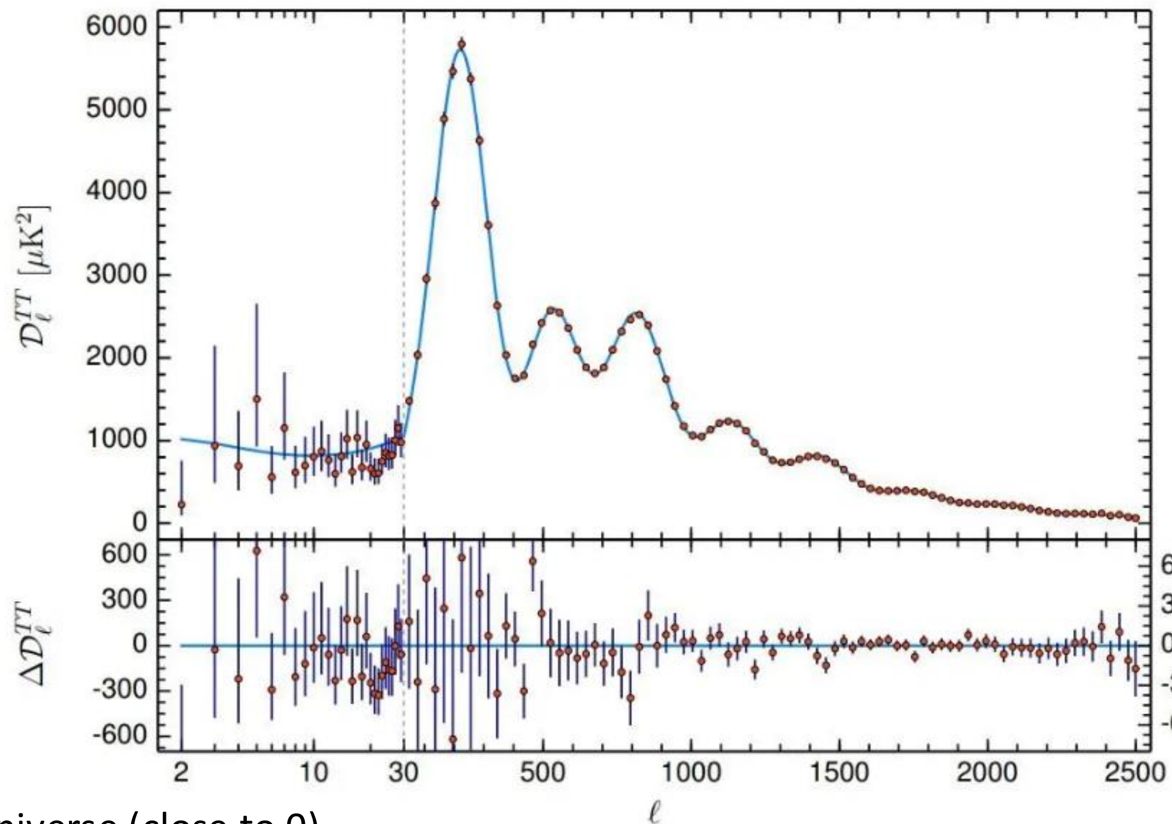
Hubble Space Telescope (2017)

James Webb Space Telescope (2022)

SMACS 0723 (z = 0.39)

An Era of "Precision Cosmology"

CMB power spectrum
(PLANCK 2018)

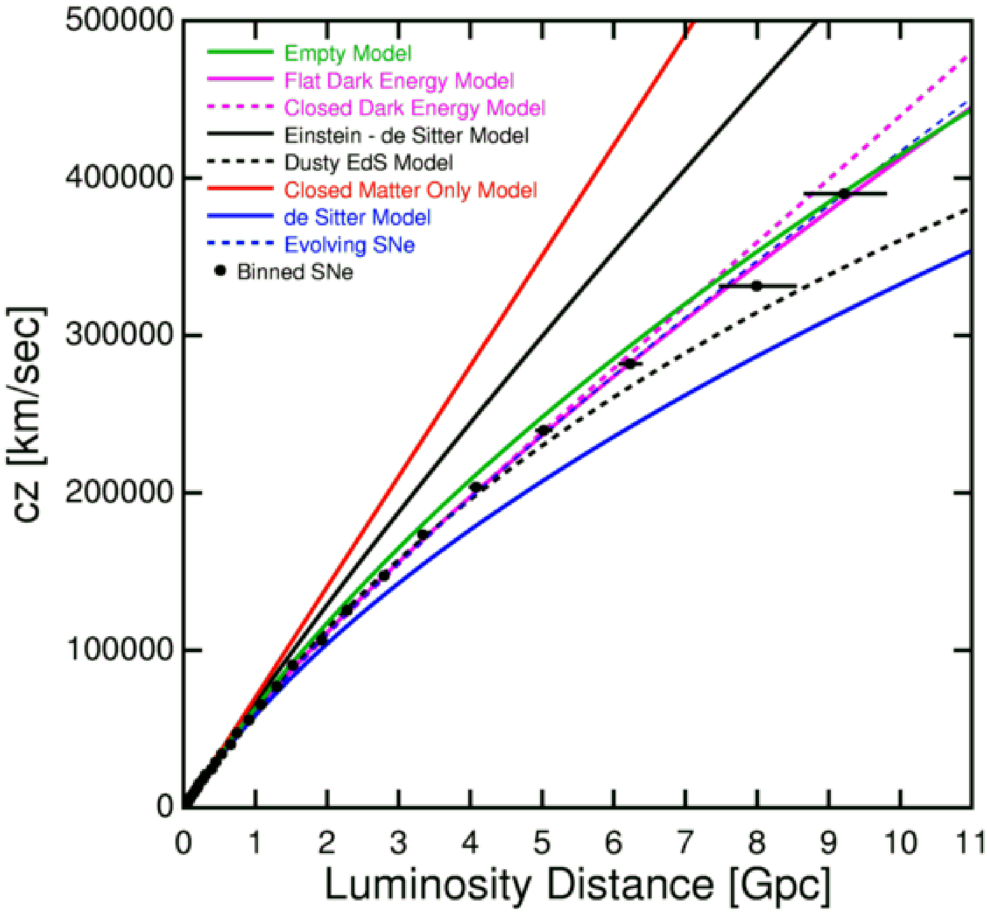


What the CMB tells us:

- The spatial curvature of the universe (close to 0).
- The ratio of densities of baryonic to dark matter (0.19).
- The ratio of baryon to photon number at recombination.
- The amplitude and tilt of the perturbation spectrum on exit from inflation.
- The size of the sound horizon at the time of last scattering.
- (Almost[#]) Nothing about the cosmological constant (or dark energy in general), as $\rho_\Lambda/\rho_m = 2 \times 10^{-9}$.

[[#]Some studies claim that the low level of scattering of CMB photons is consistent only with intervening dark energy. (See Das, Sherwin & Dunkley, *Phys. Rev. Lett.* 2011.)]

The Standard Model: ΛCDM



Wright, UCLA, 2015

Line element

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x} \cdot d\mathbf{x}$$

Scale factor

$$\dot{a}/a = H_0 \sqrt{\Omega_{m0}(a_0/a)^3 + \Omega_{r0}(a_0/a)^4 + \Omega_{\Lambda 0}}$$

where

$H_0 = 73.4$ km s^{-1} Mpc^{-1}

or 67.7 km s^{-1} Mpc^{-1}

$\Omega_{m0} \approx 0.32$

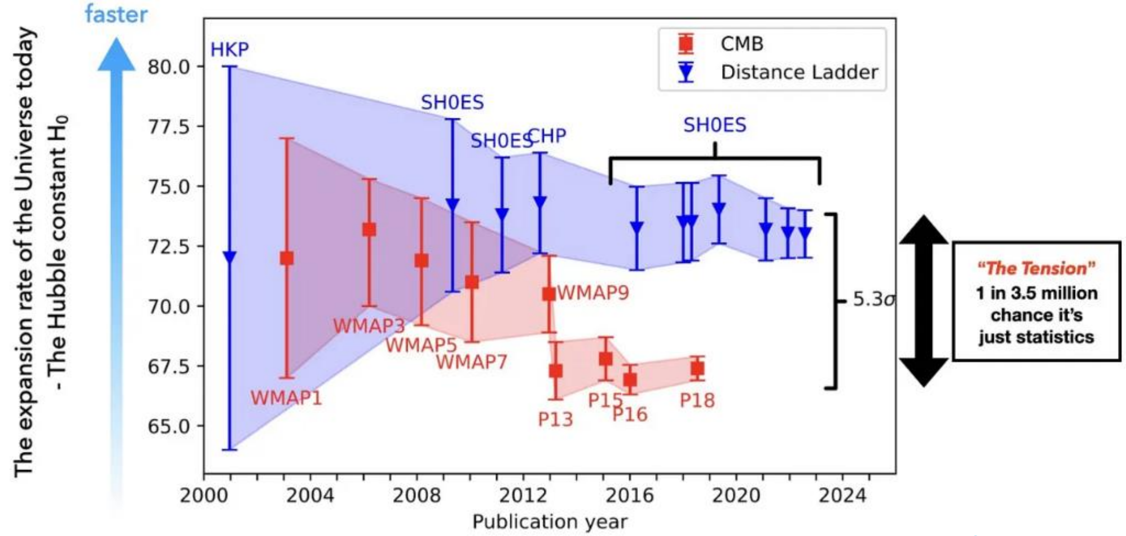
$\Omega_{\Lambda 0} \approx 0.68$

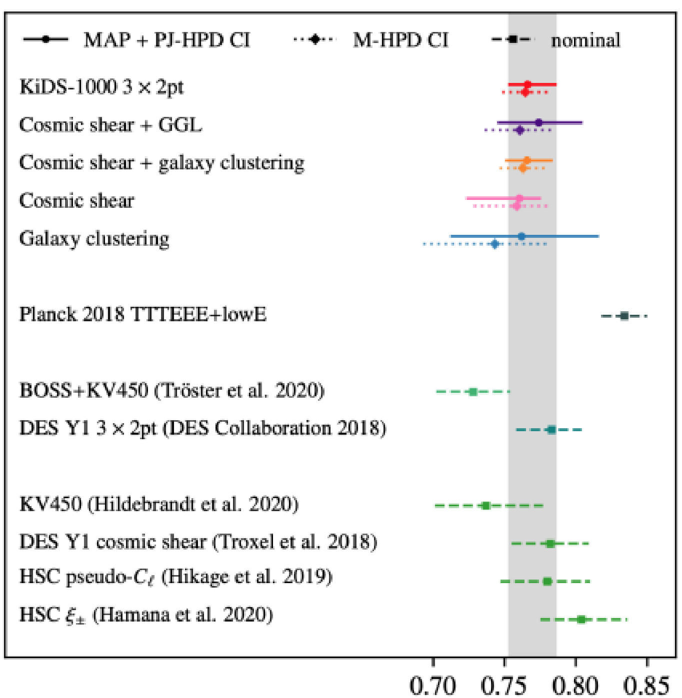
$\Omega_{r0} \sim 10^{-4}$

Standard Model "tensions"

Hubble constant ⇒

(or a "radius of sound horizon" tension)





Planck: S_8 = 0.830, Weak lensing: S_8 = 0.759

⇐ S_8 (68% error bars)

Power spectrum of density perturbations

$$S_8 = \sigma_8\sqrt{\Omega_{m0}/0.3},\ \ \sigma_8^2 = \frac{1}{2\pi^2}\int P(k)\left[\frac{3j_1(kR)}{kR}\right]^2 k^2 dk$$

$$j_1(x) = (\sin x - x\cos x)/x^2,\quad R = 8(H_0/100)\ \text{Mpc}$$

Modelling Problem: Account for SNe, Hubble and S_8, with or without dark energy

The Wilderness

Models solving the H_0 tension within 1σ, 2σ and 3σ (Planck with additional cosmological probes), from Valentino, Mena, Pan, Visinelli, Yang, Melchiorri, Mota, Riess & Silk, *Class. Quant. Grav.* 2021

tension $\leq 1\sigma$ *"Excellent models"*	tension $\leq 2\sigma$ *"Good models"*	tension $\leq 3\sigma$ *"Promising models"*
Early Dark Energy [228, 235, 240, 250]	Early Dark Energy [212, 229, 236, 263]	DE in extended parameter spaces [289]
Exponential Acoustic Dark Energy [259]	Rock 'n' Roll [242]	Dynamical Dark Energy [281, 309]
Phantom Crossing [315]	New Early Dark Energy [247]	Holographic Dark Energy [350]
Late Dark Energy Transition [317]	Acoustic Dark Energy [257]	Swampland Conjectures [370]
Metastable Dark Energy [314]	Dynamical Dark Energy [309]	MEDE [399]
PEDE [394]	Running vacuum model [332]	Coupled DM - Dark radiation [534]
Vacuum Metamorphosis [402]	Bulk viscous models [340, 341]	Decaying Ultralight Scalar [538]
Elaborated Vacuum Metamorphosis [401, 402]	Holographic Dark Energy [350]	BD-ΛCDM [852]
Sterile Neutrinos [433]	Phantom Braneworld DE [378]	Metastable Dark Energy [314]
Decaying Dark Matter [481]	PEDE [391, 392]	Self-Interacting Neutrinos [700]
Neutrino-Majoron Interactions [509]	Elaborated Vacuum Metamorphosis [401]	Dark Neutrino Interactions [716]
IDE [637, 639, 657, 661]	IDE [659, 670]	IDE [634–636, 653, 656, 663, 669]
DM - Photon Coupling [685]	Interacting Dark Radiation [517]	Scalar-tensor gravity [855, 856]
$f(\mathcal{T})$ gravity theory [812]	Decaying Dark Matter [471, 474]	Galileon gravity [877, 881]
BD-ΛCDM [851]	DM - Photon Coupling [686]	Nonlocal gravity [886]
Über-Gravity [59]	Self-interacting sterile neutrinos [711]	Modified recombination [986]
Galileon Gravity [875]	$f(\mathcal{T})$ gravity theory [817]	Effective Electron Rest Mass [989]
Unimodular Gravity [890]	Über-Gravity [871]	Super ΛCDM [1007]
Time Varying Electron Mass [990]	VCDM [893]	Axi-Higgs [991]
ΛCDM [995]	Primordial magnetic fields [992]	Self-Interacting Dark Matter [479]
Ginzburg-Landau theory [996]	Early modified gravity [859]	Primordial Black Holes [545]
Lorentzian Quintessential Inflation [979]	Bianchi type I spacetime [999]	
Holographic Dark Energy [351]	$f(\mathcal{T})$ [818]	

Simultaneous solutions to the H_0 and S_8 tensions (Yadav, Kumar, Kıbrıs, and Akarsu, 2406.18496): Λ_sCDM, New EDE, inflation with oscillations in the inflaton potential, some IDE models, sterile neutrinos with non-zero mass combined with dynamical DE, dark matter with a varying EoS, AdS-EDE with ultralight axions, some running vacuum models

Inhomogeneous Cosmologies

Abandon all symmetries $\Longrightarrow$ Trivial solution of the Modelling Problem

Mustapha, Hellaby and Ellis, *MNRAS*, 1999:
Any isotropic set of observations can be fitted by some Lemaitre-Tolman-Bondi [LTB] solution (i.e. a spherically-symmetric but inhomogeneous dust metric)

This has been done numerous times with LTB …
See Célérier, *Astron. Astrophys.*, 2000; Iguchi, Nakamura & Nakao, *Prog. Theor. Phys.*, 2002; Biswas, Mansouri & Notari, *JCAP*, 2007; Yoo, Kai & Nakao, *Prog. Theor. Phys.*, 2008; Clifton, Ferreira & Zuntz, *JCAP*, 2009; Célérier, Bolejko & Krasiński, *Astron. Astrophys.*, 2010; Vallejo & Romano, *JCAP*, 2017; etc.
(Plus "Tardis" space-time, Lavinto, Räsänen & Szybka, *JCAP*, 2013)

… and also with the Szekeres solution (P Szekeres, *Comm. Math. Phys.*, 1975; an exact anisotropic dust solution)
See Ishak, Richardson, Garred, Whittington, Nwankwo & Sussman, *Phys. Rev.* **D**, 2008

BUT these solutions either cannot be extended to $z = \infty$, or have an inhomogeneous "Big Bang time"

LTB Void models

LTB models without Λ require a local under-density (void) with a radius of the order of 3 Gpc ($z \approx 0.5$) to explain the SNe observations
See Bolejko, *PMC Phys. A*, 2008; Moss, Zibin & Scott, *Phys. Rev. D*, 2011; Redlich, Bolejko, Meyer, Lewis and Bartelmann, *Astron. Astrophys.*, 2014.

The current consensus appears to be that the Hubble tension also cannot be resolved with a local void with a radius up to 600 Mpc ($z \approx 0.1$) in a ΛLTB model: See Kenworthy, Scolnic & Riess, *Ap. J.*, 2019; Luković, Haridasu & Vittorio, *MNRAS*, 2020; Cai, Ding, Guo, Wang & Yu, *Phys. Rev. D*, 2021; Castello, Högås & Mörtsell, *JCAP*, 2022; Camarena, Marra, Sakr & Clarkson, *Class. Quantum Grav.*, 2022.

Bolejko, *Gen. Rel. Grav.*, 2009 has demonstrated that the quasi-spherical Szekeres solutions, when volume averaged, reduce to the LTB solutions.

Averaging Methods (I)

Buchert equations (Buchert, *Gen. Rel. Grav.*, 2000 & 2001)

Take an inhomogeneous line element $ds^2 = -dt^2 + g_{ij}dx^i dx^j$, assume irrotational dust, calculate volume averages $\langle \; \rangle$ over advected domains *D* of constant *t*:

$$3\ddot{a}/a = -4\pi G\,\rho_0/a^3 + \Lambda + Q$$

$$3(\dot{a}/a)^2 = 8\pi G\,\rho_0/a^3 + \Lambda - \tfrac{1}{2}\langle \mathcal{R} \rangle - \tfrac{1}{2}Q$$

where $a = (V_D/V_{D_0})^{1/3}$, $\mathcal{R}$ is the spatial Ricci scalar, and $Q = \frac{2}{3}(\langle\theta^2\rangle - \langle\theta\rangle^2) - 2\langle\sigma^2\rangle$.

- The equations are under-determined.

Räsänen, *JCAP*, 2009: Add averaged Sachs equation without null shear

$$\ddot{D} + (\dot{a}/a)\dot{D} + (\tfrac{1}{6}\langle \mathcal{R} \rangle + Q)D = 0 \qquad \text{where} \quad D = \langle D_A \rangle / a$$

- But then "anything goes" (Clifton & Hyatt, 2402.08586)
- Volume averaging effectively uses the co-moving synchronous gauge (see later)
- Volume averaging should be over the past light-cone, not a spacelike hypersurface

This has been attempted (Buchert, van Elst & Heinesen, *Gen. Rel. Grav.*, 2023)

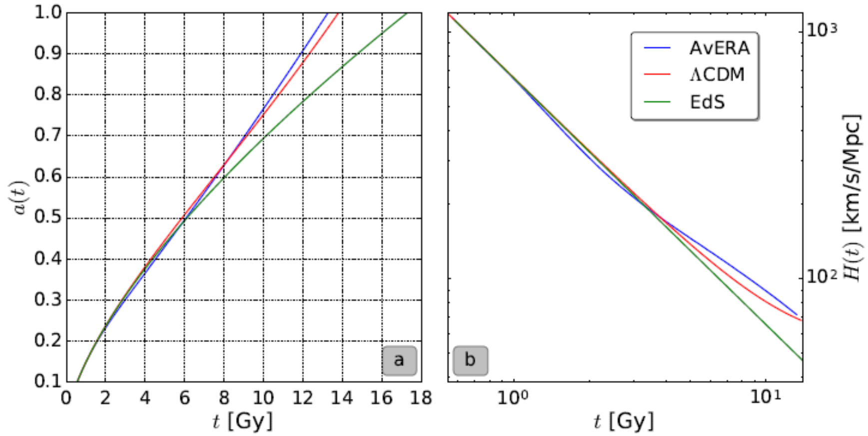
- Little more can be done without an explicit underlying spacetime

Averaging Methods (II)

"AvERA" model (Racz, Dobos, Beck, Szapudi & Csabai, *MNRAS Lett.*, 2017)

An ensemble of FLRW dust spacetimes, assuming a lognormal distribution for Ω_m with mean 1 at the time of last scattering, and adjustable variance

Scale factor



Hubble parameter

- Averaging is volume-weighted, not mass-weighted

"Timescape" model (Wiltshire, *New J. Phys.*, 2007; Wiltshire, *Phys. Rev. Lett.*, 2007; Leith, Ng & Wiltshire, *Astrophys. J.*, 2008; Wiltshire, *Int. J. Mod. Phys.*, 2008; Wiltshire, *Phys. Rev. D*, 2008; Wiltshire, *Phys. Rev. D*, 2009; Wiltshire, *Int. J. Mod. Phys.*, 2009; Smale & Wiltshire, *MNRAS*, 2011; Duley, Nazer & Wiltshire, *Class. Quantum Grav.*, 2013; Nazer & Wiltshire, *Phys. Rev. D*, 2015; Dam, Heinesen & Wiltshire, *MNRAS*, 2017; Lane, Seifert, Ridden-Harper, Wagner & Wiltshire, 2311.01438; Williams, Macpherson, Wiltshire & Stevens, 2403.15134)

An ensemble of perfect-fluid spacetimes, comprising underdense "void" regions and virialised "wall" regions

Perturbation Methods (I)

Perturbed FLRW line element (spatially flat, cold matter only):

$$ds^2 = (1+h_{tt})dt^2 + 2a(t)h_{tB}dx^B dt - a(t)^2(\delta_{BC} - h_{BC})dx^B dx^C$$

where $|h_{ab}| \ll 1$

Newtonian gauge: $h_{tt} = 2\phi(\mathrm{x}), \quad h_{BC} = 2\phi(\mathrm{x})\delta_{BC}, \quad h_{tB} = 0$

Co-moving synchronous gauge: $h_{tt} = 0, \quad h_{BC} = \frac{10}{3}\phi\,\delta_{BC} + (\frac{1}{3})^{1/3} t^{2/3} \partial_B \partial_C \phi, \quad h_{tB} = 0$

Back-reaction effects (e.g. $\delta H_0/H_0$) are small (< 1%) in the Newtonian gauge …
see Paranjape & Singh, *JCAP*, 2008; Ben-Dayan, Gasperini, Marozzi, Nugier & Veneziano, *Phys. Rev. Lett.*, 2013; Fleury, Clarkson & Maartens, *JCAP*, 2017; Adamek, Clarkson, Daverio, Durrer & Kunz, *Class. Quant. Grav.*, 2019; Macpherson, Price & Lasky, *Phys. Rev. D*, 2019

… but can be large (~15%) in the co-moving synchronous gauge
see Skarke, *Eur. Phys. J.*, 2017, Bolejko, *Phys. Rev. D*, 2018; Adamek et al., *Class. Quant. Grav.*, 2019

⇒ "Gauge Problem"

Perturbation Methods (II)

Proposed solutions to the Gauge Problem:

- "Covariant gauge-invariant" approach
(Hawking, *Astrophys. J.*, 1966; Ellis & Bruni, *Phys. Rev. D*, 1989)
- Bardeen's "gauge-invariant'' variables (Bardeen, *Phys. Rev. D*, 1980)
- Macroscopic Gravity (Zalaletdinov, *Gen. Rel. Grav.*, 1992)
- Co-moving synchronous gauge is "unphysical"
(Ishibashi & Wald, *Class. Quant. Grav.*, 2006)

The first three are ruled out by the Stewart-Walker Theorem
(Stewart & Walker, *Proc. Roy. Soc. Lon.*, 1974)

How to really eliminate gauge dependence:

τ source time → Invert; θ, ϕ observation angles → Angle Average; $x_o = [\tau_o, \mathbf{x}_o]$ observer position → Hypersurface Average

Calculate	Invert	Angle Average	Invert	Hypersurface Average
$D_A(\tau;\theta,\phi,x_o)$	$D_A(z;\theta,\phi,x_o)$	$\bar{D}_A(z;\tau_o,\mathrm{x}_o)=(4\pi)^{-1}\iint D_A\,d\Omega$	$\tau_o(\bar{H}_o,\mathrm{x}_o)$	$\langle\bar{D}_A\rangle(z;\bar{H}_o)=\lim_{V\to\infty}V^{-1}\iiint\bar{D}_A\,dV$
$z(\tau;\theta,\phi,x_o)$	$H_o(\theta,\phi,x_o)=\lim_{z\to 0}(dD_A/dz)^{-1}$	$\bar{H}_o(\tau_o,\mathrm{x}_o)=(4\pi)^{-1}\iint H_o\,d\Omega$	$\bar{D}_A(z;\bar{H}_o,\mathrm{x}_o)$	where $V=\iiint dV$, $dV=[-\det(g^{(3)})\vert_{\bar{H}_o}]^{1/2}d^3\mathrm{x}_o$

Green: Gasperini, Marozzi, Nugier & Veneziano, *Phys. Rev. Lett.*, 2013; Fleury, Clarkson & Maartens, *JCAP*, 2017

Blue: Magi & Yoo, *JCAP*, 2022 (but with age of universe in place of H_o)

Yellow: Unpublished

2009 Retrospective

“[Cosmological theory] seems rather reminiscent of the Ptolemaic epicycles for describing the solar system, with more and more complex mechanisms added in as the simpler ones failed to describe the phenomena.

“The physically most conservative approach is to assume no unusual dark energy or exotic interacting fields, but rather that an inhomogeneous geometry might be responsible for the observed apparent acceleration; this should be seriously considered as an alternative.”

G F R Ellis, *J. Phys.: Conf. Ser.*, 2009

Picture Credits

Wilderness:
https://science.howstuffworks.com/environmental/earth/geology/largest-desert-in-world.htm

COBE/WMAP/Planck: https://www.forbes.com/sites/startswithabang/2018/07/19/how-the-planck-satellite-changed-our-view-of-the-universe/

Hubble/JWST:
https://petapixel.com/2022/07/11/comparing-hubble-to-james-webb-the-difference-in-detail-is-astounding/

CMB:
https://www.forbes.com/sites/startswithabang/2018/07/19/how-the-planck-satellite-changed-our-view-of-the-universe/

Redshift-distance:
https://astro.ucla.edu/~wright/sne_cosmology.html

Hubble tension:
https://medium.com/@glennborchardt/science-magazine-the-universes-puzzlingly-fast-expansion-may-defy-explanation-cosmologists-13464b839e3d

AvERA:
Rácz et. al, "Concordance cosmology without dark energy", *MNRAS Lett*, **469**, L1 (2017)

George Ellis:
https:// creativeprocess.info/books-writers/george-ellis-mia-funk-gl7mp