Galactic dynamics beyond dark matter

ASGRG 30 Years Conference

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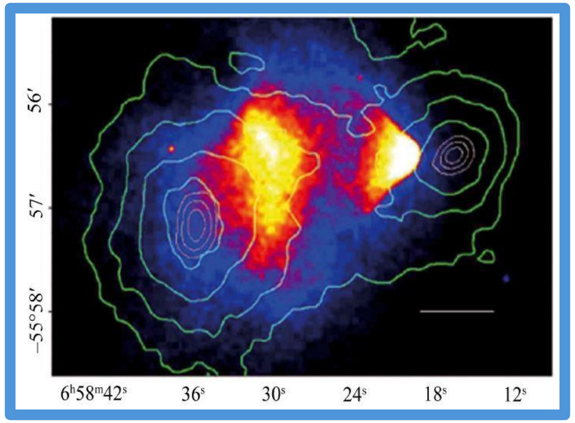
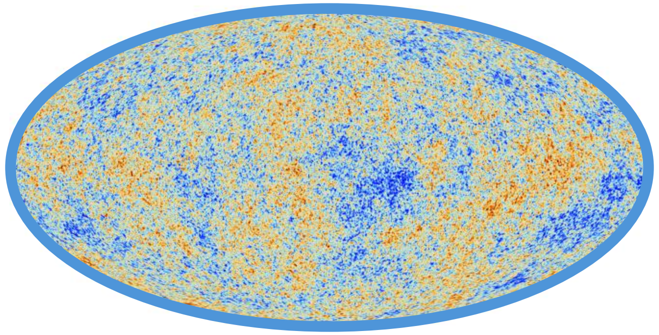
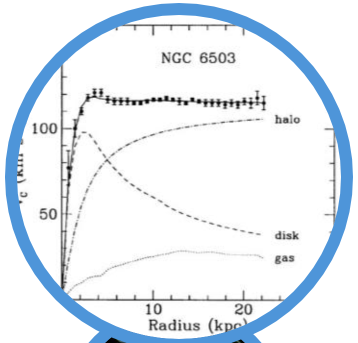
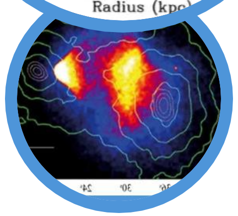


The Missing Mass Problem

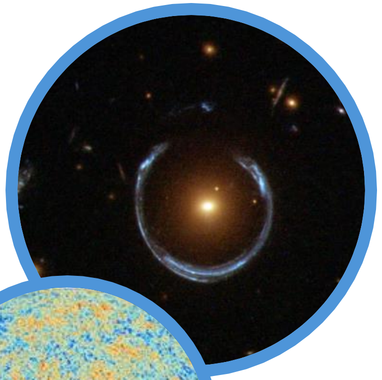


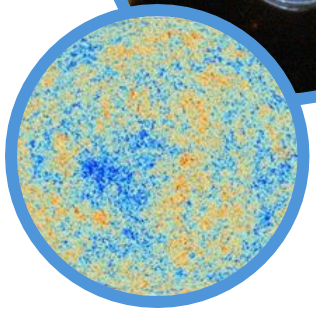
The dark matter (DM) hypothesis remarkably explains a plethora of astrophysical and cosmological observations.



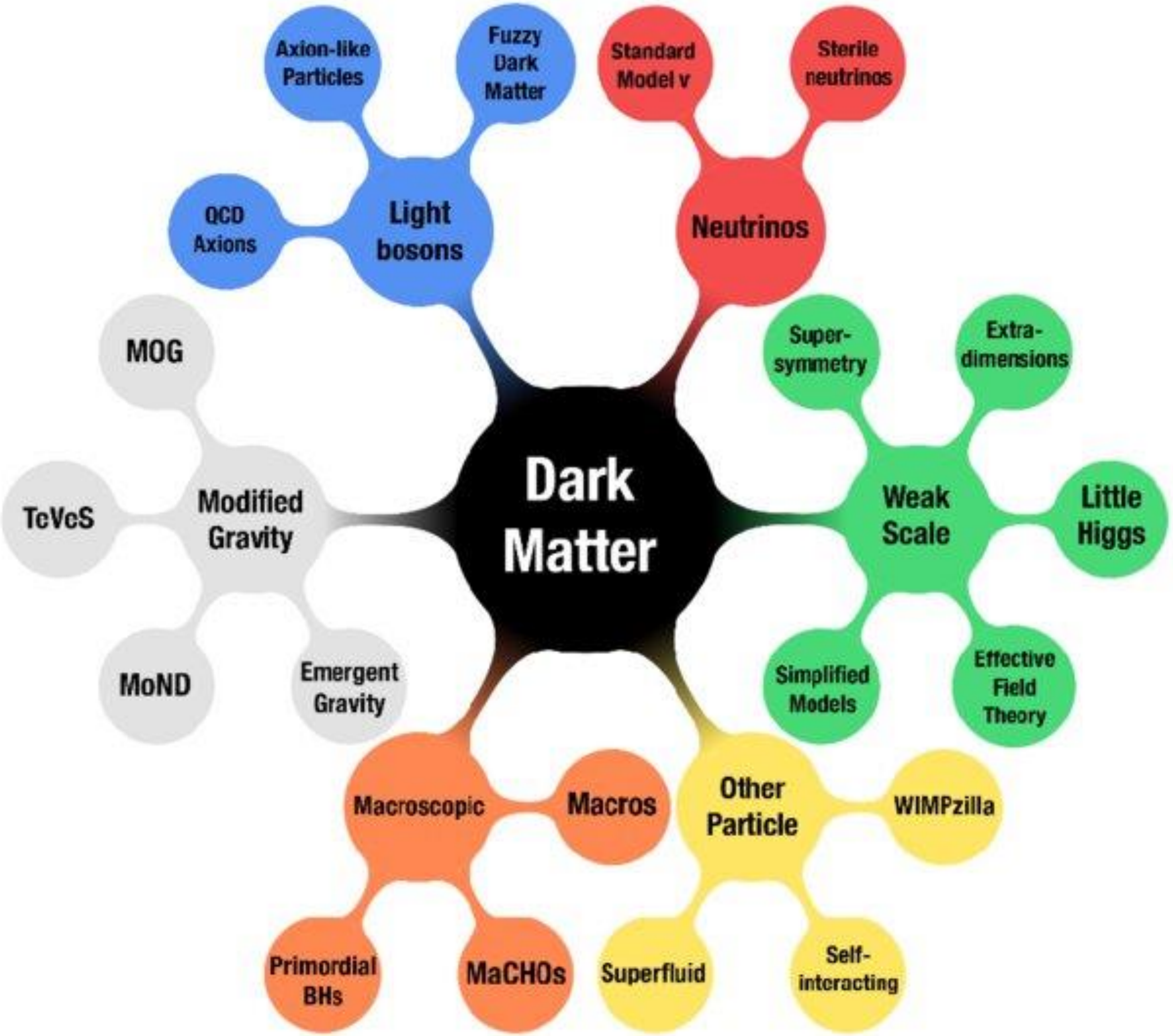


1. Disc galaxy rotation curves;
2. Virial estimation of galaxies' velocities in galaxy clusters;
3. Strong gravitational lensing by galaxies and clusters of galaxies;
4. Temperature of hot gasses in galaxies and clusters;
5. Bullet clusters' dynamics and observed gravitational lensing;
6. CMB anisotropies;
7. Large-scale structures' formation,
8. Etc. etc.



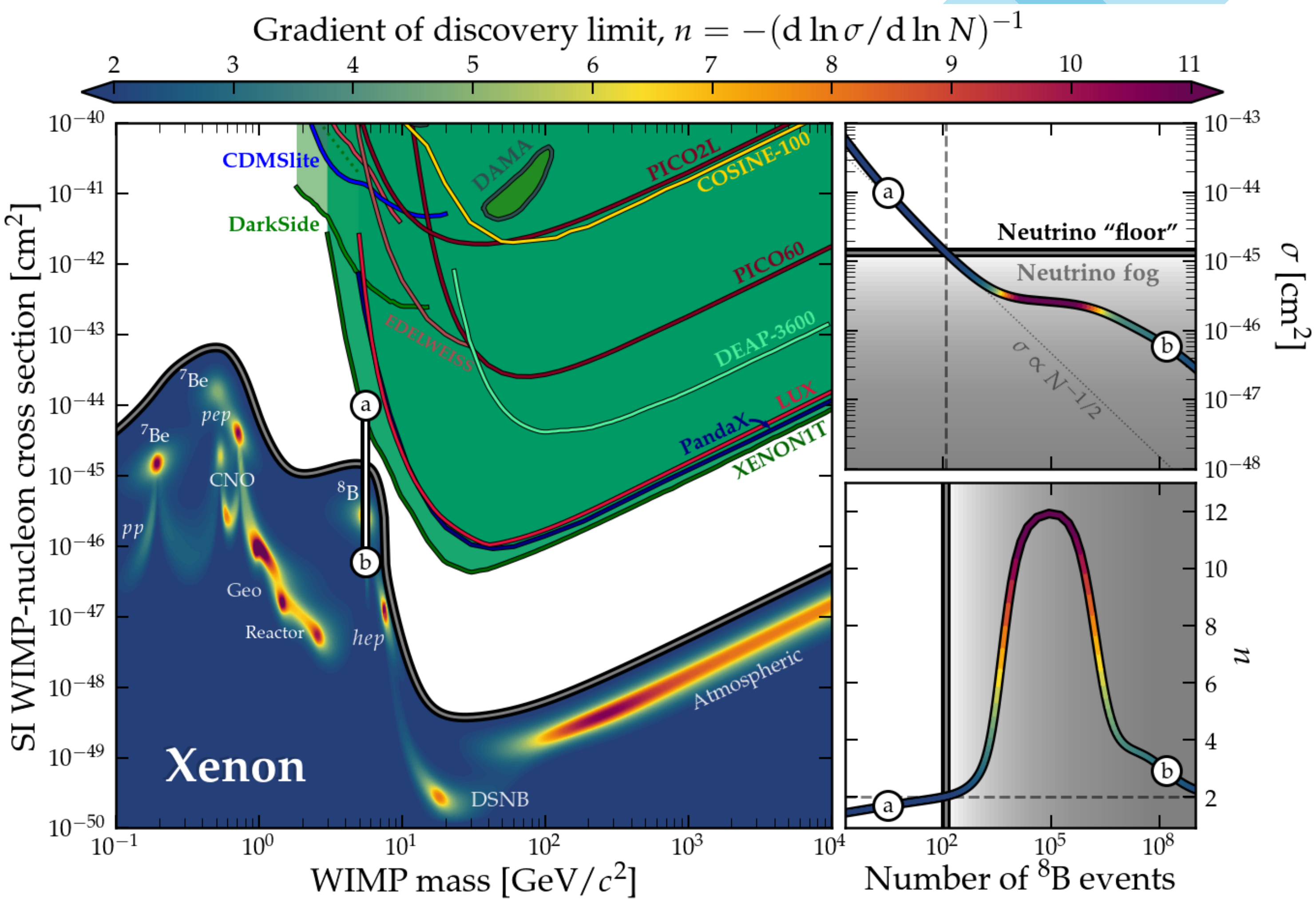


- We do not know what constitutes DM.
- We are swimming in an ocean of possibilities:
  1. WIMPS?
  2. MACHOS?
  3. Axions?
  4. Others?
- That said, we are certainly looking for it ...



N. Yan, Theoretical and Natural Science, **19**, 90-101 (2023)

- Unfortunately, after more than three decades, **every direct detection experiment for DM search has given null results**.
- The new generation of detectors will be completed in the coming decade (e.g., SABRE)
- However, WIMPS direct detection experiments are about to hit the **Neutrino fog** .

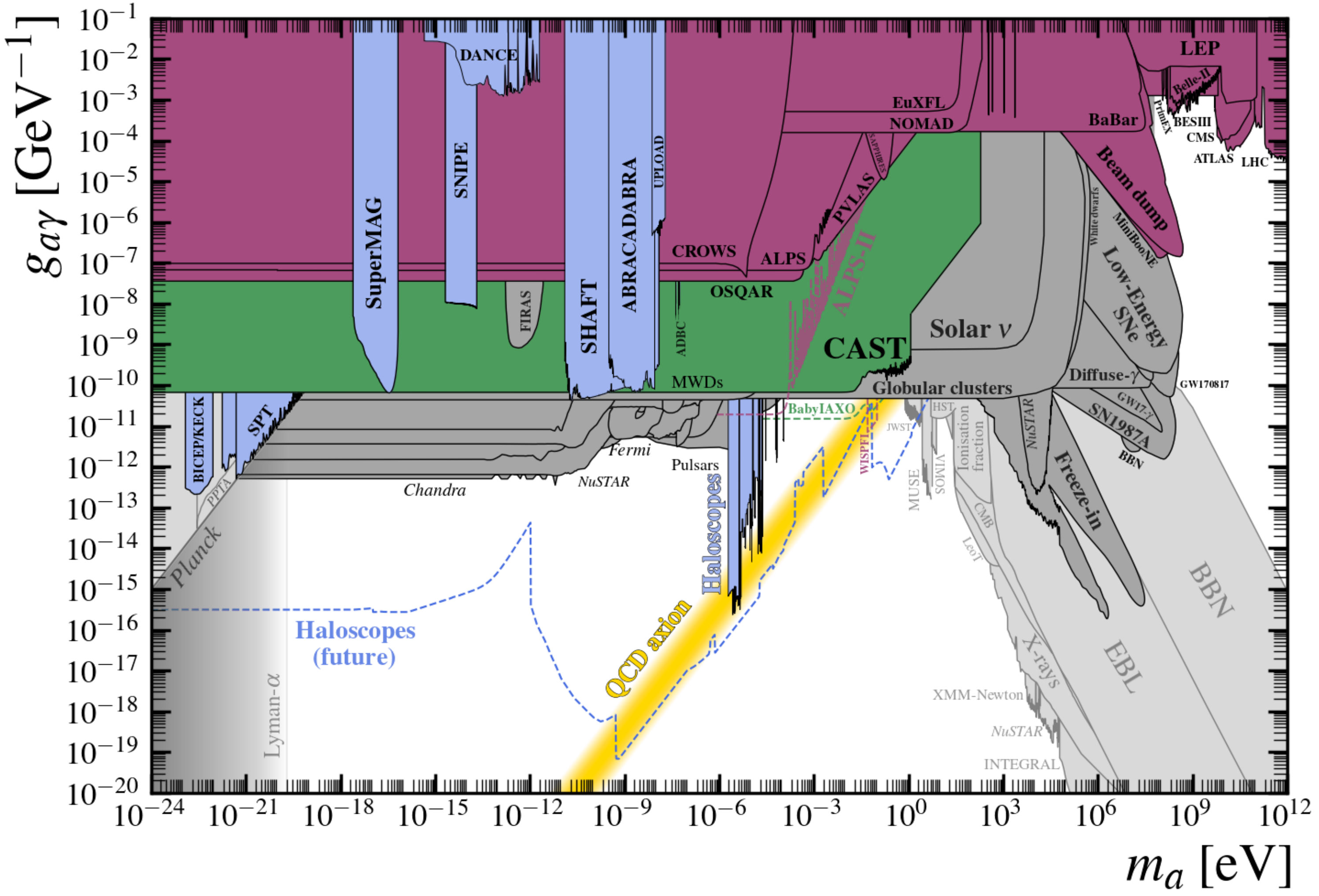


Ciaran A. J. O'Hare, Phys. Rev. Lett. **127**, 251802 (2021)

- Experiments are thoroughly searching for **QCD Axions and axions-like** DM particles.

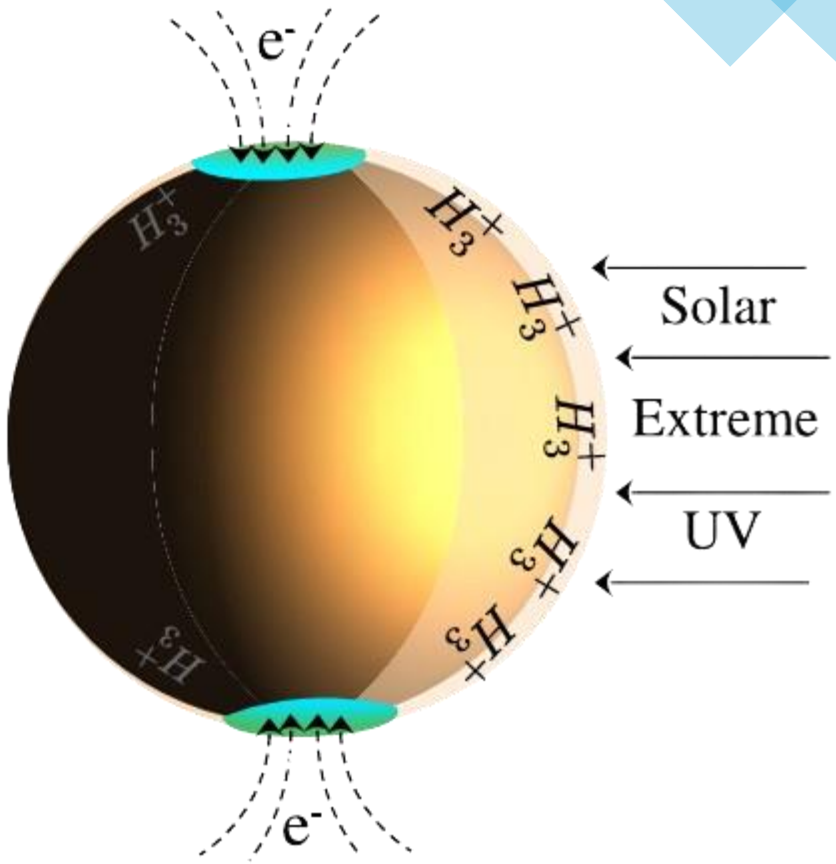
- Future Haloscopes will cover a wide area of the phase-space of interest, hopefully giving positive results soon.

- However, up to now, direct detection of ultra-light DM still eludes the scientific community



Ciaran A. J. O’Hare, FIPS Workshop (2022)

- We are branching out, and with better telescopes new avenues for direct detection may open up (e.g., using gravitational waves).
- However, even using Jupiter as a planet-size detector for self-interacting DM has only returned null results.
- Have we been just unlucky, or **is there something more that we are missing?**



C. Blanco, and R. K. Lane, Phys. Rev. Lett. **132**, 261002 (2024)

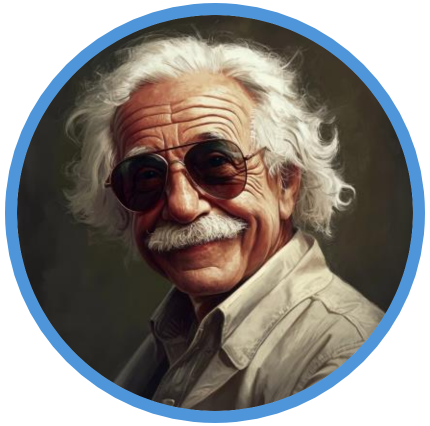
Some observations:

1. All the missing mass phenomena **(MMP) are gravitational in nature**.

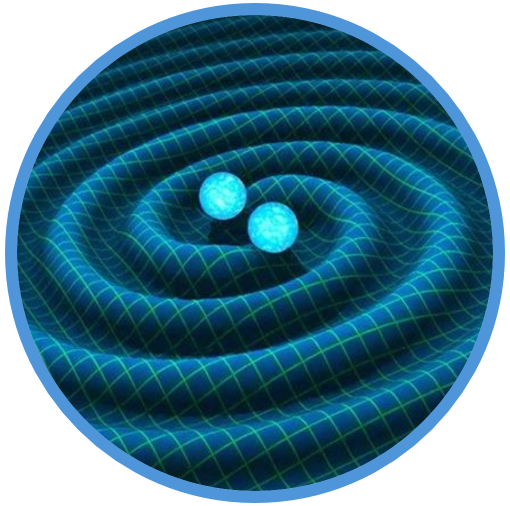
2. All the MMP systems are in a low-energy regime, i.e., weak gravitational potential and nonrelativistic velocities.

3. To study the MMP we thus employ **Newtonian perturbation over a fixed Minkwoksi** (or FLRW) background.

4. Could it be that this conventional approach is leaving something out?



General relativity: a solution?



- Conventionally, general relativity (GR) is believed to reduce to Newtonian gravity in the low-energy limit.

- Post-Newtonian corrections give contributions to the dynamics of order $(v/c)^n$ with $n \geq 2$.

- However, **this widespread belief is fundamentally wrong**, e.g., gravitational waves.

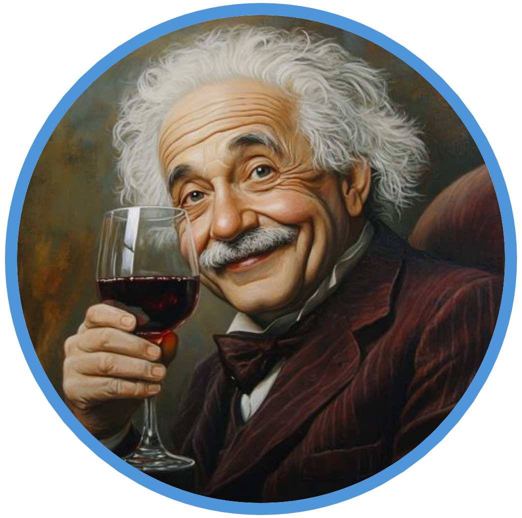
- **GR allows for non-Newtonian behaviour in the low-energy limit.** How is this possible?

- Einstein’s field equations **(EFE) are non-linear,** in contrast to the Poisson equation.

- The GR gravitational field carries energy and angular momentum, in contrast to the non-dynamical Newtonian field.

- From a physics standpoint this means that the self-interaction of matter and geometry via Einstein's equations defines **regional backgrounds with their own quasilocal energy and angular momentum content**.

- As far as mathematics is concerned, enter noncommutativity, stage left…



These procedures do not commute!

1. Assume small metric perturbations, i.e., $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/c^2$ with $h_{\mu\nu}/c^2 \ll 1$;
2. Deduce the EFE;
3. Find Newton, i.e.,
4. Obtain Newtonian gravity:

$$\Delta\Phi = 4\pi G\rho_M + \mathcal{O}\left(v^2/c^2\right)$$

Conventional approach

$\neq$

1. Write the general metric which respects the symmetries of the system;
2. Deduce the EFE;
3. Take the nonrelativistic limit, i.e., expand to the lowest order in v/c;
4. Obtain first-order correction to Newton:

$$\Delta\Phi + \ldots = 4\pi G\rho_M + \mathcal{O}\left(v^2/c^2\right)$$

Our novel approach

Quasilocal Newtonian limit of general relativity and galactic dynamics

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A new Newtonian limit of general relativity is established for stationary axisymmetric gravitationally bound differentially rotating matter distributions with internal pressure. The self-consistent coupling of quasilocal gravitational energy and angular momentum leads to a modified Poisson equation. The coupled equations of motion of the effective fluid elements are also modified, with quasilocal angular momentum and frame–dragging leading to novel dynamics. The solutions of the full system reproduce the phenonomenology of collisionless dark matter for disc galaxies, offering an explanation for their observed rotation curves. Halos of abundant cold dark matter particles are not required.

- We start with the building blocks of our theory and physical system, i.e., the generalized Lewis–Papapetrou–Weyl metric and the energy-momentum tensor. Notice, this metric is reasonably valid for timescales of $10^7 \lesssim t \lesssim 10^9$.

Metric & EMT

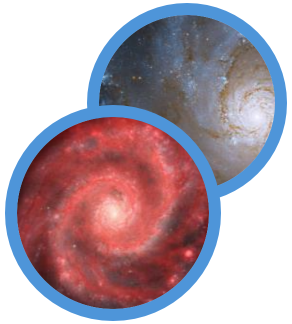
$$ds^2 = -c^2 e^{2\Phi(r,z)/c^2} (\mathrm{d}t + A(r,z)\,\mathrm{d}\phi)^2 + e^{-2\Phi(r,z)/c^2} \left[W(r,z)^2\, \mathrm{d}\phi^2 + e^{2k(r,z)/c^2} (\mathrm{d}r^2 + \mathrm{d}z^2)\right]$$

$$T^{\mu\nu} = (\rho_M(r,z) + p(r,z)/c^2)\, U^\mu U^\nu + p(r,z) g^{\mu\nu}$$

$$U^\mu \partial_\mu = (-H(r,z))^{-1/2} (\partial_t + \Omega(r,z)\,\partial_\phi)$$

$$H = -e^{2\Phi/c^2}(1 + A\,\Omega)^2 + e^{-2\Phi/c^2} W^2\, \Omega^2/c^2$$

- We can now move on to calculate the EFE and the equations of motion (EOM).



EFE

$$\Phi^{,a}{}_{,a} + \frac{W^{,a}\Phi_{,a}}{W} + c^4\frac{A^{,a}A_{,a}}{2W^2}e^{4\Phi/c^2} = 4\,\pi\,G e^{2(k-2\Phi)/c^2}\left[\left(\rho_M + \frac{p}{c^2}\right)\frac{(1+A\Omega)^2 e^{2\Phi/c^2} + c^{-2}W^2\Omega^2 e^{-2\Phi/c^2}}{-H} + 2\frac{p}{c^2}\right],$$

$$A^{,a}{}_{,a} - \frac{W^{,a}A_{,a}}{W} + \frac{4}{c^2}\Phi^{,a}A_{,a} = \frac{16\,\pi\,G}{c^4}W^2\Omega\frac{1+A\Omega}{H}\left(\rho_M + \frac{p}{c^2}\right)e^{2(k-2\Phi)/c^2},$$

$$W^{,a}{}_{,a} = \frac{16\,\pi\,G}{c^4}p\,,$$

$$W_{,rr} - W_{,zz} + \frac{2}{c^2}(k_{,z}W_{,z} - k_{,r}W_{,r}) + \frac{2W}{c^4}\left(\Phi_{,r}^2 - \Phi_{,z}^2\right) + \frac{c^2}{2W}e^{4\Phi/c^2}\left(A_{,z}^2 - A_{,r}^2\right) = 0\,,$$

$$\frac{c^2}{2W}e^{4\Phi/c^2}A_{,z}\,A_{,r} - W_{,rz} - \frac{2W}{c^4}\Phi_{,r}\Phi_{,z} + \frac{1}{c^2}(k_{,z}W_{,r} - k_{,r}W_{,z}) = 0\,,$$

$$\frac{1}{c^2}\left(\Phi^{,a}{}_{,a} - k^{,a}{}_{,a}\right) - \frac{1}{2W}W^{,a}{}_{,a} + \frac{1}{c^2}\frac{W^{,a}\Phi_{,a}}{W} - \frac{1}{c^4}\Phi^{,a}\Phi_{,a} + \frac{c^2}{4W^2}e^{4\Phi/c^2}\left(A_{,z}^2 + A_{,r}^2\right) = \frac{4\,\pi\,G}{c^2}e^{2(k-2\Phi)/c^2}\left(\rho_M - \frac{p}{c^2}\right),$$

$$H\frac{p_{,a}}{\rho_M}e^{2(k-\Phi)/c^2} = \left[\Phi_{,a} + (c^2A_{,a} + 2A\Phi_{,a})\Omega + (c^2AA_{,a} + A^2\Phi_{,a})\Omega^2\right]e^{2\Phi/c^2} + \Omega^2\left(W^2\frac{\Phi_{,a}}{c^2} - WW_{,a}\right)e^{-2\Phi/c^2}$$

EOM

- Following our novel approach, we must now take the nonrelativistic limit for our system of equations.
- We thus impose:
  1. Small velocities, $v \ll c$;
  2. Weak gravitational potential, $\Phi \sim v^2$;
  3. Nonrelativistic frame-dragging, i.e., small $\chi := -g_{t\phi}/g_{\phi\phi}$;
  4. Small pressure, $p \sim \rho_M v^2$.

- We also define:

$v_K := r\,\Omega\,,$ → rotation curve velocity

$v_D := r\,\chi\,,$ → dragging velocity

↓

$$\mathrm{L}_D := r\, v_D$$

→ We then find

$$H = -1 + \frac{v_K^2}{c^2} - 2\frac{v_K}{c}\frac{v_D}{c} + \frac{2\,\Phi}{c^2} + \mathcal{O}\left(v^4/c^4\right)$$

$$A = r\,v_D/c^2 + \mathcal{O}(v^3/c^3)$$

↓ and we assume

$$W(r,z) = r + \mathcal{O}\left(v^4/c^4\right)\,,$$

EFE

$$\Delta\Phi + \frac{1}{2r^2}||\vec{\nabla}Ł_D||^2 = 4\pi G\rho_M + \mathcal{O}\left(v^2/c^2\right) ,$$

$$\hat{\Delta}Ł_D = 0 + \mathcal{O}\left(v^2/c^2\right) ,$$

$$k_{,r} = \frac{1}{4r}\left(Ł_{D,z}^2 - Ł_{D,r}^2\right) + \mathcal{O}\left(v^2/c^2\right) ,$$

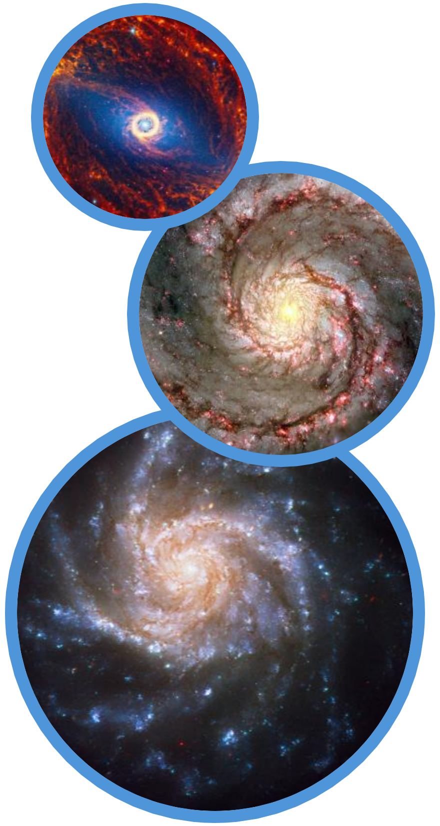
$$k_{,z} = -\frac{1}{2r}\left(Ł_{D,z}\, Ł_{D,r}\right) + \mathcal{O}\left(v^2/c^2\right) ,$$

EOM

$$-\frac{p_{,r}}{\rho_M} = \Phi_{,r} + \Omega\, Ł_{D,r} - v_K^2/r + \mathcal{O}\left(v^2/c^2\right) ,$$

$$-\frac{p_{,z}}{\rho_M} = \Phi_{,z} + \Omega Ł_{D,z} + \mathcal{O}\left(v^2/c^2\right) .$$

Extra, first-order terms which modify Newtonian dynamics



- To investigate the quasilocal Newtonian limit of galactic dynamics we begin by assuming a “realistic” baryonic matter distribution, i.e,

$$\rho_B = \rho_b + \rho_d$$

where

$$\rho_b(R) = \frac{3R_b^2 M_b}{4\pi \left(R^2 + R_b^2\right)^{5/2}}\,,$$

→ Plummer’s mass bulge profile.

and $\rho_d(r, z) = \Sigma(r) Z(z)$, with

$$\Sigma(r) = \frac{M_d}{2\pi r_d^2} e^{-r/r_d}\,,$$

$$Z(z) = \frac{1}{2z_d} e^{-|z|/z_d}$$

Exponential disc mass profile.

We employ the common thin disc approximation.

- We then create a mock rotation curve for a Milky Way-like galaxy following the Newtonian + DM paradigm and assuming an isothermal halo, i.e. :

$$\rho_{DM}(R) = \frac{\rho_{DM0}}{1 + (R/R_{DM})^2} \,.$$

To solve the quasilocal equations we define

$$\Delta\Phi_D = \frac{||\vec{\nabla}\text{Ł}_D||^2}{2r^2} \,.$$

We can obtain and solve the equations

$$\vec{\nabla}\text{Ł}_D \cdot \left[\vec{\nabla}\text{Ł}_D - 2\vec{\nabla}(r v_K)\right] + 2r\left(v_K^2 - v_B^2\right)_{,r} = 0 \,.$$

Its solution allows us to check the dragging velocity magnitude: it must be nonrelativistic.

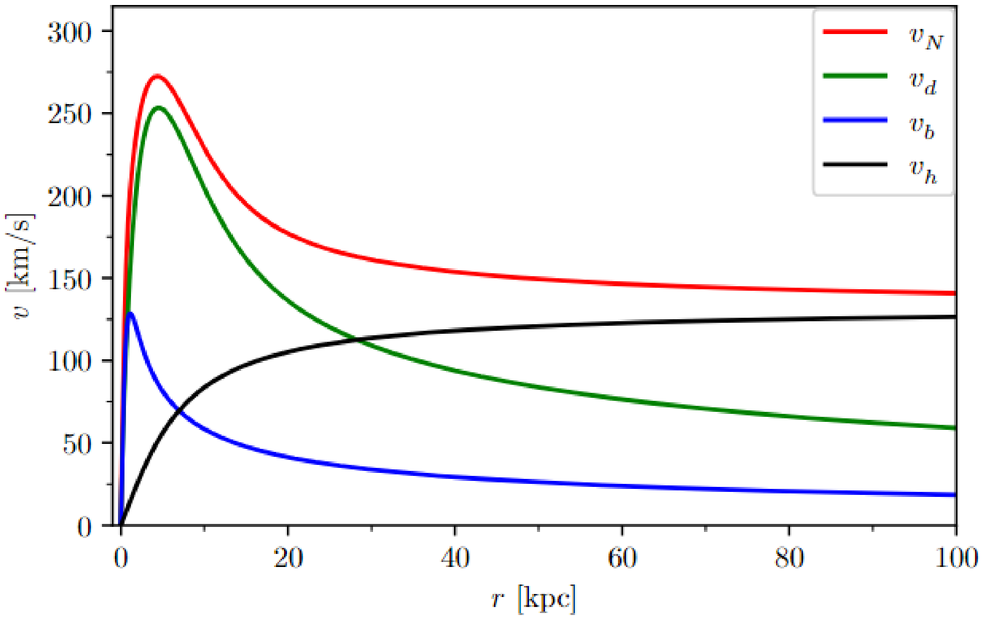


FIG. 1. Conventional Newtonian rotation curve of the thin disc, $v_K(r, 0)$, and the respective $v_b(r, 0)$, $v_d(r)$ and $v_h(r, 0)$, for a Milky Way-like galaxy with $M_b = 0.8 \cdot 10^{10}$ M$_\odot$, $R_b = 0.8$ kpc, $M_d = 8.1 \cdot 10^{10}$ M$_\odot$, $r_d = 2.1$ kpc, $R_{DM} = 5.69$ kpc and $\rho_{DM0} = 6.77 \cdot 10^{-22}$ kg/m^3.

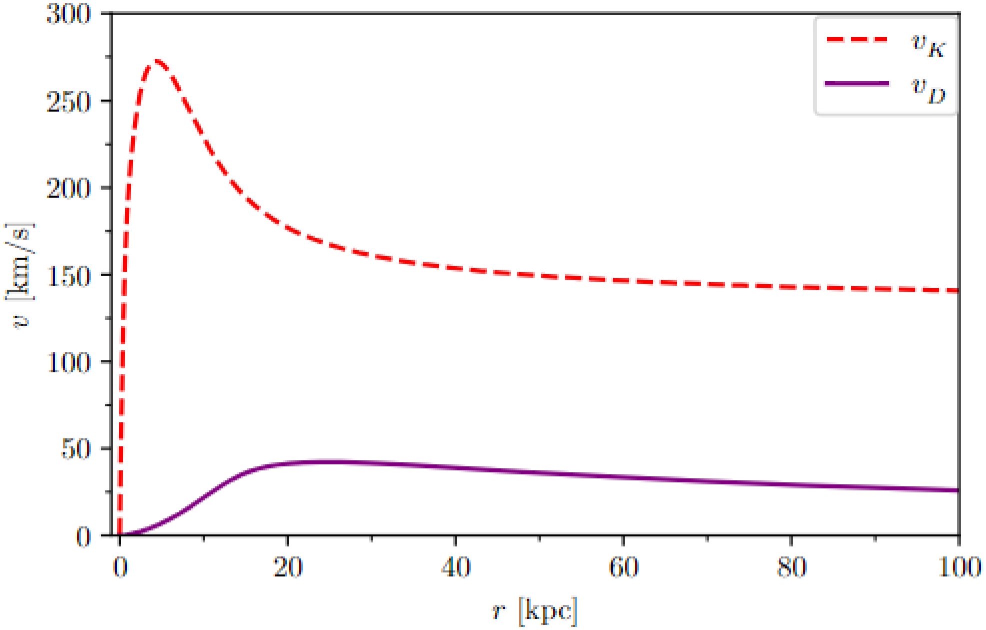


FIG. 2. The profile for $v_D(r, 0)$ needed to support the simulated rotation curve, $v_K(r, 0)$, for a Milky Way-like galaxy with $M_b = 0.8 \cdot 10^{10}$ M$_\odot$, $R_b = 0.8$ kpc, $M_d = 8.1 \cdot 10^{10}$ M$_\odot$ and $r_d = 2.1$ kpc. Here, the galaxy is composed exclusively of baryonic matter, i.e, $\rho_{DM} = 0$.

CONCLUSIONS

- **The quasilocal Newtonian limit of galaxy dynamics exhibits a phenomenology equivalent to DM.**
- The two hypotheses have the **same degree of predictiveness.**
- The quasilocal energy and angular momentum of the regional spacetime can replace DM in sustaining the rotation curves.
- **Such effects if confirmed point to a change in the interpretation of what we deem particulate DM.**
- Still, a loooooong way to go…

FUTURE WORK: THEORY

1. Investigate the **stability of disc galaxies** in the quasilocal Newtonian limit. In the conventional DM models the halo stabilizes the galaxy, can the quasilocal energy content do the same? **This will be a theoretical check for our theory.**

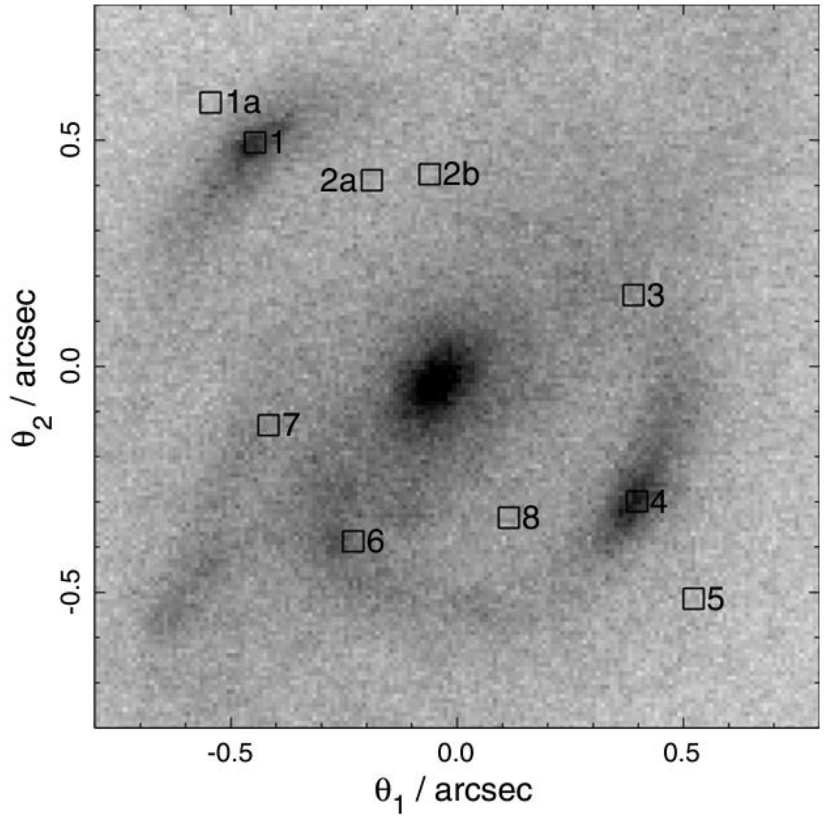
2. Develop a **formalism for gravitational lensing** in the quasilocal Newtonian limit. We can still use the thin lens approximation, but the dragging will influence the lensing.

FUTURE WORK: OBSERVATIONAL TESTS

- Couple dynamical data (i.e., rotation curves) and gravitational lensing observations to **break the model degeneracies** and stress-test the theory.

- Current lens candidates (not great):
  1. B1933+503;
  2. SDSSJ2141−0001*.

- Upcoming optical survey will easily allow for this test, e.g., LSST is predicted to see hundreds of thousands of galaxy lenses (see Thomas E. Collett 2015 ApJ **811** 20).



S. H. Suyu et al. 2012, ApJ, **750**, 10 (B1933+503)

FUTURE WORK: OBSERVATIONAL TESTS

- Statistical analysis of the rotational velocity distributions of **co-rotating and counter-rotating stars** in disc galaxies (**GAIA DR3** might be just what we need...perhaps).

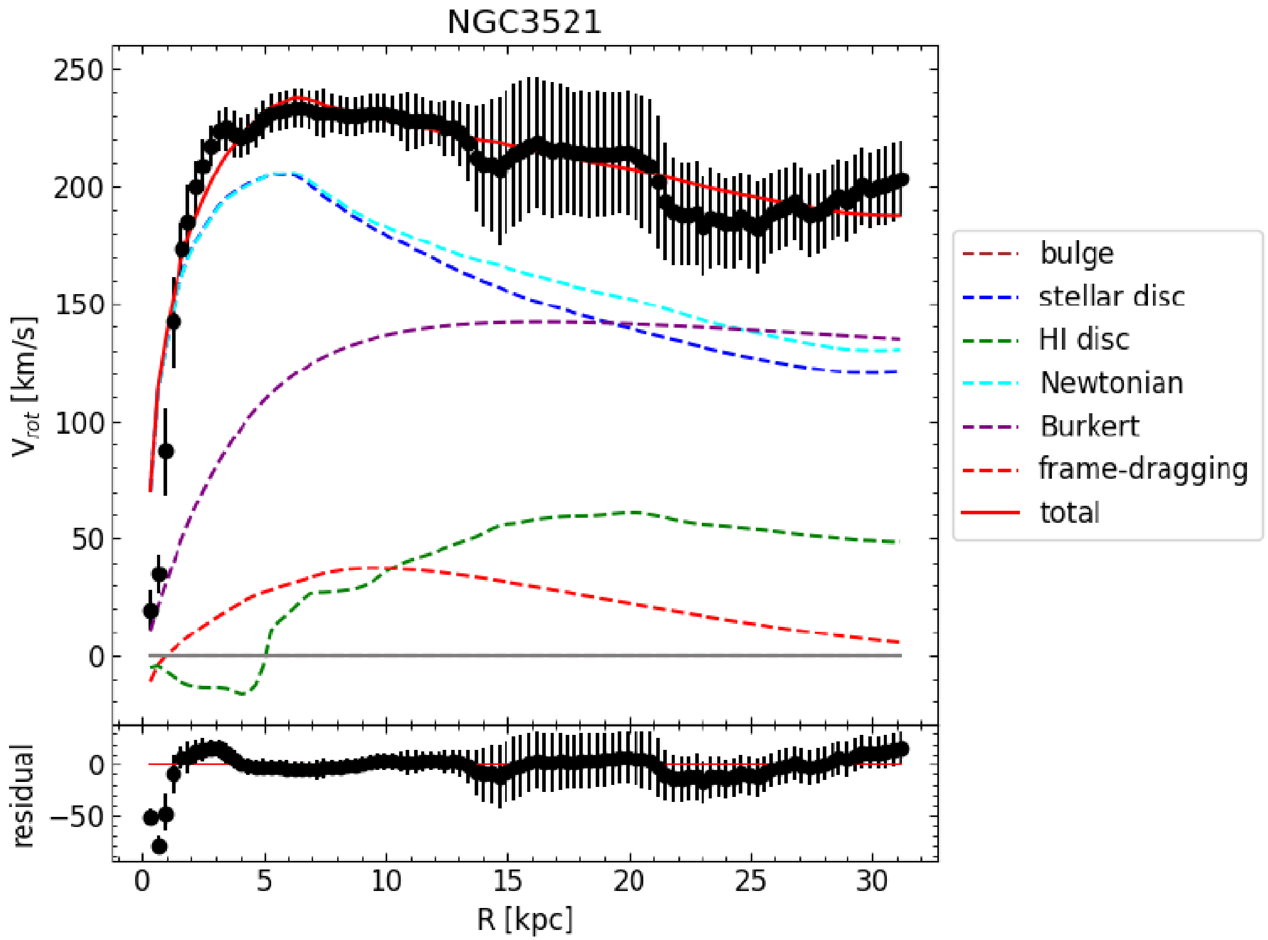
- Due to the spacetime dragging velocity, $v_D(r, z)$, we expect a difference between the two distributions in the quasilocal Newtonian limit (contrary to the Newtonian case).

- We find :

  $\Delta v_K(r, 0) = (r\, v_D(r, 0))_{,r} \approx 30\,\mathrm{km/s}$ at 30 kpc.

- Limitations:
  1. The counter-rotating part should be minimal;
  2. Correct selection of the population of stars analysed;
  3. High precision required.

FUTURE WORK: A SNEAK PEEK



Hessman et al (in preparation)…fitting of real data!

Thank you for your attention!

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BACK-UP SLIDES

Effective galactic dark matter: first order general relativistic corrections

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- The EFE for the spacetimes considered can be cast in the **nonlinear gravitomagnetic form** (in a sense...)

$$\begin{cases} \nabla \cdot \mathbf{G} = \mathbf{G}^2/c^2 + \mathbf{H}^2/2c^2 - 4\pi G\rho \\ \nabla \times \mathbf{G} = 0 \\ \nabla \cdot \mathbf{H} = -\mathbf{G} \cdot \mathbf{H}/c^2 \\ \nabla \times \mathbf{H} = 2\mathbf{G} \times \mathbf{H}/c^2 - 16\pi G\mathbf{j}/c^3 \end{cases}$$

where

$$\mathbf{G} := -\nabla\Phi$$
$$\mathbf{H} := e^{\Phi/c^2}(\nabla \times \mathbf{A})$$
$$j^\mu := -T^{\mu\nu}U_\nu/c^2$$

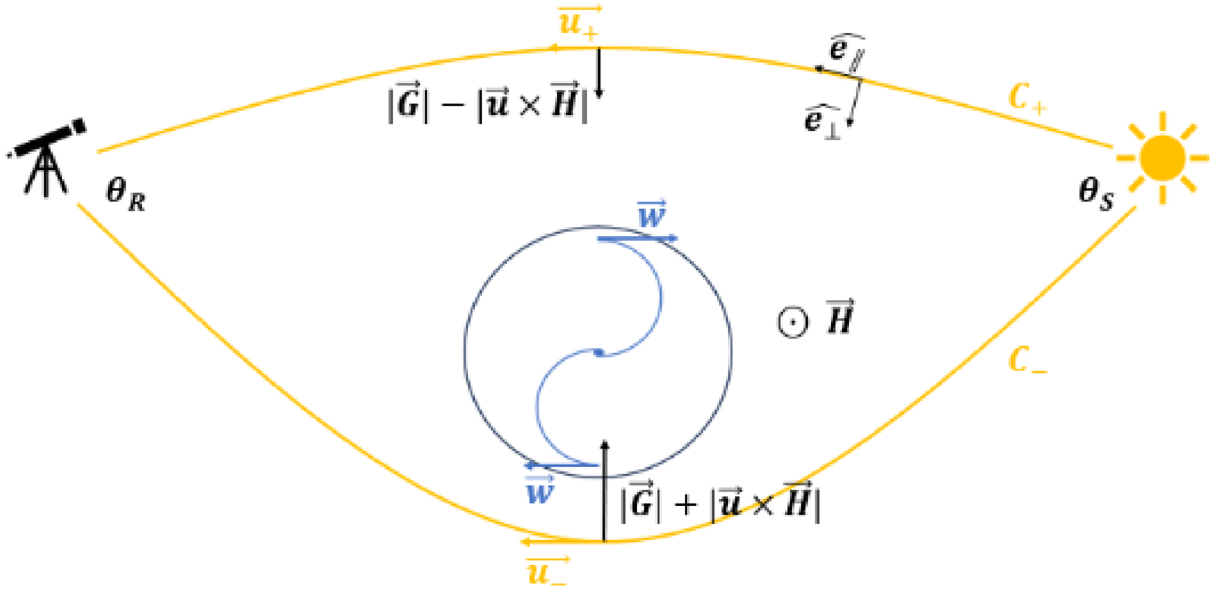

Figure 3. A sketch of gravitational lensing due to a dragging metric. The asymmetry between the co-rotating and counter-rotating side is stressed, as it is the reason why such lensing results to be bigger than the zero dragging case. Here, the galaxy is shown viewed from below (i.e., negative z)

$$\Delta\theta_{dragging} \approx -2c \int H_{,r}(r_0)\Delta r(\lambda) d\lambda \approx \frac{2}{c}\int \left(\frac{v_D}{r^2} - \frac{v_{D,r}}{r} - v_{D,rr}\right) \Delta r(\lambda)\, \mathrm{d}\lambda \; > 0\,.$$

We find a **first-order**, **positive contribution** from the draggin vortex to gravitational lensing.